



# RoadMap Of Mathematics

*Lesson Plan*

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## Overview & Purpose

"Roadmap Math" serves as a comprehensive guide that outlines the key components of mathematics. It provides a structured map of various mathematical fields, offering a brief description of each area to help students and educators understand the breadth and depth of mathematics. The book is designed to be a reference tool, summarizing the essential topics and their relationships.

### Purpose:

The primary purpose of "Roadmap Math" is to offer a clear and concise summary of the major areas of mathematics. It aims to help students identify areas of interest and understand how different mathematical concepts interconnect. This book is ideal for educators planning curriculum, students preparing for advanced studies, and anyone interested in gaining a high-level understanding of mathematical topics.

### Education Standards

"Roadmap Math" adheres to educational standards such as the Common Core State Standards (CCSS) and the National Council of Teachers of Mathematics (NCTM). The content is organized to support standard curriculum frameworks, ensuring that it is relevant and useful for K-12 education as well as introductory college courses.

### Objectives

**General Objectives:**

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- To provide a succinct overview of fundamental mathematical concepts.
  - To illustrate the interconnectedness of various mathematical fields.
  - To serve as a reference guide for educators and students.
  - To prepare students for more detailed studies in specific areas of mathematics.

### **Specific Objectives:**

- Identify and describe various number systems.
- Summarize key concepts in algebra, geometry, and calculus.
- Outline advanced mathematical topics such as topology and differential equations.
- Provide examples to illustrate each concept briefly.

## **Materials Needed**

- "Roadmap Math" textbook
- Basic calculator
- Geometry toolkit (optional for visualization)
- Access to online resources for supplementary information

# **Cataloging the components of comprehensive mathematics**

## **1. Number Systems**

- Natural Numbers (N)

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- Whole Numbers (W)
  - Integers (Z)
  - Rational Numbers (Q)
  - Real Numbers (R)
  - Complex Numbers (C)
  - Algebraic Numbers
  - Transcendental Numbers
  - Binary Numbers
  - Octal Numbers
  - Decimal Numbers
  - Hexadecimal Numbers
  - p-adic Numbers: A system of number theory that extends the concept of integers.

## 2. Structures

### Algebra

- Foundations
- Algebraic expressions
- Linear equations and inequalities
- Graphing lines and slope
- Systems of equations
- Expressions with exponents
- Quadratics and polynomials
- Equations and geometry
- Abstract Algebra:
  - Groups
  - Rings
  - Fields

- Modules
- Algebras
- Galois Theory: Study of symmetries in algebraic equations.
- Boolean Algebra: Algebraic structure used in logic and computer science.

### Linear Algebra

- Vectors and spaces
- Matrix transformations
- Alternate coordinate systems

### Number Theory

- Divisibility
- Prime numbers
- Diophantine equations
- Modular arithmetic
- Cryptography
- **Analytic Number Theory**: Uses analysis to solve number theory problems.

### Combinatorics

- Permutations
- Combinations
- Graph Theory
- Ramsey Theory

### Order Theory

- Partially ordered sets
- Lattices

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## Logic

- Propositional logic
- Predicate logic
- Set theory
- Model theory
- Proof theory
- Computability theory

## Information Theory

- Entropy
- Coding theory

## 3. Spaces

### Geometry

- Lines
- Angles
- Shapes
- Triangles
- Quadrilaterals
- Coordinate plane
- Area and perimeter
- Volume and surface area
- Pythagorean theorem
- Transformations
- Congruence
- Similarity
- Trigonometry
- Circles
- Analytic geometry

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- Geometric constructions
  - **Non-Euclidean Geometry:** Hyperbolic and elliptic geometry.
  - **Algebraic Geometry:** Study of zeros of multivariate polynomials.
  - **Projective Geometry:** Study of geometric properties invariant under projection.
  - **Discrete Geometry:** Study of geometric objects and their combinatorial properties.
  - Miscellaneous

### Trigonometry

- Right triangles & trigonometry
- Trigonometric functions
- Non-right triangles & trigonometry
- Trigonometric equations and identities

### Topology

- Point-set topology
- Algebraic topology
- Differential topology
- **Topological Spaces:** Concepts like open and closed sets, continuity.

### Fractal Geometry

- Self-similarity
- Fractal dimensions

### Differential Geometry

- Curves and surfaces
- Riemannian geometry
- Connections and curvature

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## Measure Theory

- Sigma-algebras
- Measures and integration

## 4. Changes

### Calculus

- **Differential Calculus:**
  - Limits and continuity
  - Derivatives: definition and basic rules
  - Derivatives: chain rule and other advanced topics
  - Applications of derivatives
  - Analyzing functions
  - Parametric equations, polar coordinates, and vector-valued functions
- **Integral Calculus:**
  - Integrals
  - Differential equations
  - Applications of integrals
  - Parametric equations, polar coordinates, and vector-valued functions
  - Series
- **Multivariable Calculus:** Calculus in higher dimensions.
- **Vector Calculus:** Field theory, divergence, and curl.

### Differential Equations

- Ordinary Differential Equations (ODEs)
- Partial Differential Equations (PDEs)

### Dynamical Systems

- Stability theory
- Bifurcation theory
- Chaos theory

### **Complex Analysis**

- Analytic functions
- Contour integration
- Series expansions

### **Functional Analysis**

- Banach and Hilbert spaces
- Operators on infinite-dimensional spaces

### **Fourier Analysis**

- Fourier series and transforms

### **Numerical Analysis**

- Numerical solutions to equations
- Error analysis

### **Probability and Statistics**

- Probability theory
- Descriptive statistics
- Inferential statistics
- Stochastic processes

### **Mathematical Physics**

- Quantum mechanics
- General relativity

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- Statistical mechanics

### **Computational Mathematics**

- Algorithm theory
- Computational complexity
- Computer algebra systems

### **Applied Mathematics**

- Operations research
  - Financial mathematics
  - Biomathematics
  - Engineering mathematics
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## 1. Number Systems

### Natural Numbers (N)

**Definition:** Natural numbers are the set of positive integers used for counting. They start from 1 and go on indefinitely.

**Example:** 1, 2, 3, 4, 5, ...

**Use Case Example:** Counting the number of apples in a basket.

### Whole Numbers (W)

**Definition:** Whole numbers include all natural numbers along with zero.

**Example:** 0, 1, 2, 3, 4, 5, ...

**Use Case Example:** Calculating the total number of items in a list that includes no items, such as an empty basket.

### Whole Numbers (W)

**Definition:** Whole numbers include all natural numbers along with zero.

**Example:** 0, 1, 2, 3, 4, 5, ...

**Use Case Example:** Calculating the total number of items in a list that includes no items, such as an empty basket.

### Integers (Z)

**Definition:** Integers consist of all whole numbers and their negative counterparts.

**Example:** -3, -2, -1, 0, 1, 2, 3, ...

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**Use Case Example:** Representing temperature changes, where temperatures can fall below zero (e.g.,  $-5^{\circ}\text{C}$ ).

### **Rational Numbers (Q)**

**Definition:** Rational numbers are numbers that can be expressed as the quotient or fraction of two integers, where the denominator is not zero.

**Example:**  $1/2$ ,  $-3/4$ ,  $5$ ,  $0.75$ , ...

**Use Case Example:** Dividing a pizza into equal parts, where each part is a fraction of the whole pizza.

### **Real Numbers (R)**

**Definition:** Real numbers include all rational and irrational numbers, encompassing all points on the number line.

**Example:**  $3$ ,  $-1.5$ ,  $\sqrt{2}$ ,  $\pi$ , ...

**Use Case Example:** Measuring lengths and distances, which can be whole numbers, fractions, or irrational numbers like  $\sqrt{2}$  (the length of the diagonal of a unit square).

### **Complex Numbers (C)**

**Definition:** Complex numbers have a real part and an imaginary part and are expressed in the form  $a + bi$ , where "i" is the imaginary unit ( $\sqrt{-1}$ ).

**Example:**  $2 + 3i$ ,  $-1 + 4i$ ,  $0 - 2i$ , ...

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**Use Case Example:** Electrical engineering calculations often use complex numbers to represent impedance in AC circuits.

### Algebraic Numbers

**Definition:** Algebraic numbers are roots of non-zero polynomial equations with integer coefficients.

**Example:** Solutions to the equation  $x^2 - 2 = 0$  are  $\sqrt{2}$  and  $-\sqrt{2}$ .

**Use Case Example:** Finding the roots of polynomial equations in algebra.

### Transcendental Numbers

**Definition:** Transcendental numbers are numbers that are not roots of any non-zero polynomial equation with integer coefficients.

**Example:**  $\pi$  (pi),  $e$  (Euler's number).

**Use Case Example:** Calculating the circumference of a circle ( $2\pi r$ ) or exponential growth ( $e^x$ ).

### Binary Numbers

**Definition:** Binary numbers are expressed in the base-2 numeral system, using only 0 and 1.

**Example:** 1010 (binary for decimal 10), 1101 (binary for decimal 13).

**Use Case Example:** Computer data storage and processing, where all data is represented in binary.

### Octal Numbers

**Definition:** Octal numbers are expressed in the base-8 numeral system, using digits from 0 to 7.

**Example:** 10 (octal for decimal 8), 17 (octal for decimal 15).

**Use Case Example:** File permissions in Unix/Linux systems are often represented in octal.

### Decimal Numbers

**Definition:** Decimal numbers are expressed in the base-10 numeral system, the standard system for denoting integer and non-integer numbers.

**Example:** 10, 0.5, -3.14, 7.89, ...

**Use Case Example:** Everyday counting, monetary transactions, and measurements.

### Hexadecimal Numbers

**Definition:** Hexadecimal numbers are expressed in the base-16 numeral system, using digits from 0 to 9 and letters A to F.

**Example:** 1A (hexadecimal for decimal 26), FF (hexadecimal for decimal 255).

**Use Case Example:** Color codes in web design (e.g., #FFFFFF for white) and memory addresses in computing.

### p-adic Numbers

**Definition:** p-adic numbers are a system of number theory that extends the concept of integers for a given prime  $p$ , providing a different way to measure "closeness."

**Example:** In the 3-adic system, the series  $1 + 3 + 3^2 + 3^3 + \dots$  converges to a 3-adic number.

**Use Case Example:** p-adic numbers are used in number theory and cryptography for certain types of algorithms and proofs.

## 2. Structures

### Algebra

Algebra is a branch of mathematics that deals with symbols and the rules for manipulating those symbols. These symbols represent quantities without fixed values, known as variables. Algebra includes the study of algebraic expressions, equations, and inequalities. It also covers the operations and relations, the concepts of polynomial functions, and how to solve various equations, both linear and quadratic. Algebra forms the foundation for advanced studies in mathematics and is widely applied in science, engineering, medicine, and economics.

### Foundations

- **Definition:** Basic concepts of algebra, including variables, constants, and the operations performed on them.
- **Example:** Understanding that in the expression  $3x+23x + 23x+2$ , 3 is a coefficient,  $x$  is a variable, and 2 is a constant.
- **Use Case Example:** Simplifying algebraic expressions such as  $2(x+3)=2x+6$   
 $2(x + 3) = 2x + 6$   
 $2(x+3)=2x+6$

### Algebraic Expressions

- **Definition:** Combinations of variables, numbers, and operators (such as +, -, \*, /).
- **Example:**  $4x^2-3x+7$   
 $4x^2 - 3x + 7$
- **Use Case Example:** Formulating the cost of items bought in terms of variables, like  $C=3x+5y$   
 $C = 3x + 5y$  for buying  $x$  items of one type and  $y$  of another.

### Linear Equations and Inequalities

- **Definition:** Equations of the first degree, and inequalities representing the relationship between expressions.
- **Example:** Linear Equation:  $2x+3=7$ , Linear Inequality:  $x-4 \leq 2x-4$
- **Use Case Example:** Solving for the amount of ingredients needed to balance a recipe, like finding  $x$  in  $2x+3=7$ .

### Graphing Lines and Slope

- **Definition:** Plotting linear equations on a coordinate plane and determining the steepness (slope) of the line.
- **Example:** The equation  $y=2x+1$  graphs as a line with a slope of 2 and y-intercept at 1.
- **Use Case Example:** Determining the rate of change in distance over time in a speed vs. time graph.

### Systems of Equations

- **Definition:** A set of two or more equations with the same variables.
- **Example:**  $\begin{cases} y = x + 2 \\ y = 2x - 1 \end{cases}$
- **Use Case Example:** Solving for the point where two lines intersect, like finding the common solution in the system above.

### Expressions with Exponents

- **Definition:** Algebraic expressions that include exponents.
- **Example:**  $3x^2 - 4x + 5$
- **Use Case Example:** Calculating compound interest,  $A = P(1 + \frac{r}{n})^{nt}$

### Quadratics and Polynomials

- **Definition:** Polynomial expressions of degree 2 (quadratics) and higher.
- **Example:** Quadratic:  $ax^2+bx+c=0$   
Polynomial:  $3x^4-5x^3+2x-1$
- **Use Case Example:** Modeling projectile motion with a quadratic equation  $y=-4.9t^2+vt+h$

## Equations and Geometry

- **Definition:** Using algebra to solve geometric problems.
- **Example:** Finding the equation of a circle:  $(x-h)^2+(y-k)^2=r^2$
- **Use Case Example:** Determining the dimensions of shapes given certain constraints, like finding the radius of a circle.

## Linear Algebra

Linear Algebra is the branch of mathematics concerning linear equations, linear functions, and their representations through matrices and vector spaces. It includes studying vector spaces, linear transformations, eigenvalues, and eigenvectors. Linear algebra is essential in numerous areas of mathematics and science, including physics, computer science, and statistics. It is used in various applications such as computer graphics, signal processing, and solving systems of linear equations.

## Vectors and Spaces

- **Definition:** Vectors are quantities with both magnitude and direction, and spaces are sets of vectors.
- **Example:** A vector in 2D space:  $\mathbf{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$
- **Use Case Example:** Representing forces in physics as vectors.

## Matrix Transformations

- **Definition:** Using matrices to perform linear transformations.
- **Example:** A rotation matrix  $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$  rotates vectors by angle  $\theta$ .
- **Use Case Example:** Computer graphics transformations like scaling and rotating images.

## Alternate Coordinate Systems

- **Definition:** Different ways to describe geometric locations, such as polar coordinates.
- **Example:** Polar coordinates  $(r, \theta)$  describe a point's distance and angle from the origin.
- **Use Case Example:** Navigation systems use alternate coordinate systems like GPS (latitude and longitude).

## Group Theory

Group Theory studies algebraic structures known as groups. A group is a set equipped with a single operation that combines any two elements to form a third element, satisfying four conditions called the group axioms: closure, associativity, identity, and invertibility. Group theory is fundamental to abstract algebra and has applications in many fields, including physics, chemistry, and computer science, particularly in studying symmetry and solving polynomial equations.

- **Definition:** A group is a set  $G$  with an operation  $*$  such that for all  $a, b, c \in G$ , the following hold: closure, associativity, identity element, and inverses.
- **Example:** The set of integers under addition  $(\mathbb{Z}, +)$ .

- **Use Case Example:** Cryptography algorithms like RSA rely on group theory.

### Combinations (Combinatorics)

Combinatorics is the branch of mathematics dealing with combinations, permutations, and the enumeration of elements in sets. It involves studying counting methods, graph theory, and the properties of finite structures. Combinatorics has applications in computer science for algorithm analysis, cryptography, and network design, as well as in statistical physics, biology, and optimization problems.

- **Definition:** The study of counting and arrangements of objects.
- **Example:** Finding the number of ways to choose 3 objects from 5:  
 $\binom{5}{3} = 10$
- **Use Case Example:** Determining the number of possible password combinations.

### Number Theory

Number Theory is the study of integers and integer-valued functions. It includes topics such as divisibility, prime numbers, modular arithmetic, and the solutions of equations in integers (Diophantine equations). Number theory has profound applications in cryptography, coding theory, and even in understanding the structure of the natural numbers. It also involves more advanced topics like analytic number theory and algebraic number theory.

- **Definition:** The study of integers and integer-valued functions.
- **Example:** The prime factorization of 28 is  $2^2 \times 7$ .
- **Use Case Example:** Coding theory and encryption, where prime numbers play a crucial role.

### Order Theory

Order Theory is the branch of mathematics that studies various types of ordered sets. An ordered set is a set equipped with a relation that defines an arrangement of its elements. This includes concepts such as lattices, posets (partially ordered sets), and total orders. Order theory is applied in many areas of mathematics and computer science, particularly in data structure design, sorting algorithms, and the theory of computation.

- **Definition:** The study of ordered sets and their properties.
- **Example:** The set of real numbers with the usual order  $(\mathbb{R}, \leq)$ .
- **Use Case Example:** Database indexing, where elements are sorted and queried efficiently.

## Logic

Logic in mathematics is the study of formal systems of reasoning. It includes propositional logic, predicate logic, and modal logic, focusing on the principles of valid inference and demonstration. Logic forms the foundation of mathematical proof, algorithm design, and computer science, especially in areas such as programming languages, artificial intelligence, and database theory.

**Example:** Propositional logic uses symbols to represent statements and logical connectives (like AND, OR, NOT) to form complex expressions and deduce new statements.

## Boolean Algebra

Boolean Algebra is a branch of algebra that deals with boolean values, true and false. It involves operations such as AND, OR, and NOT, which correspond to intersection, union, and complement in set theory. Boolean algebra is fundamental to digital electronics and computer science,

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particularly in designing circuits, programming, and understanding logical statements and search algorithms.

**Example:** In Boolean algebra, the expression  $A \text{ AND } (B \text{ OR } C)$  represents a logical operation that can be used to simplify digital circuits.

### Abstract Algebra

Abstract Algebra studies algebraic structures such as groups, rings, fields, modules, and vector spaces. It extends the familiar arithmetic of numbers to more complex systems, examining how these structures interact and the rules that govern them. Abstract algebra is foundational for many branches of mathematics and theoretical computer science and has applications in cryptography, coding theory, and quantum mechanics.

**Example:** A ring is an algebraic structure consisting of a set equipped with two binary operations (addition and multiplication) that generalize the arithmetic of integers.

### Information Theory

Information Theory studies the quantification, storage, and communication of information. It involves concepts such as entropy, data compression, and error correction. Information theory is crucial in telecommunications, computer science, and cryptography, providing the mathematical foundations for data transmission and storage technologies.

**Example:** Shannon's entropy is a measure of the uncertainty in a random variable, used to determine the optimal length of codes for data compression.

### Set Theory

Set Theory is the branch of mathematical logic that studies sets, which are collections of objects. It forms the basis for most of contemporary mathematics, providing a foundational framework for mathematical analysis, topology, and many other fields. Set theory includes concepts such as unions, intersections, subsets, and power sets.

**Example:** The set of all natural numbers  $\{1, 2, 3, \dots\}$  is an infinite set, and the set of all subsets of a given set is called its power set.

## 3. Spaces

### Geometry

Geometry is the branch of mathematics that studies the sizes, shapes, properties, and dimensions of objects and spaces. It encompasses various subfields including Euclidean geometry (dealing with flat surfaces), non-Euclidean geometry (dealing with curved surfaces), and analytic geometry (using algebra to describe geometric principles).

### Lines

- **Definition:** A straight one-dimensional figure extending infinitely in both directions.
- **Example:** The equation of a line in a 2D plane:  $y=2x+3$
- **Use Case Example:** Determining the shortest distance between two points.

### Angles

- **Definition:** Formed by two rays with a common endpoint, measured in degrees or radians.
- **Example:** A right angle is 90 degrees.
- **Use Case Example:** Calculating angles in construction to ensure structures are built correctly.

### Shapes

- **Definition:** The form of objects like circles, triangles, squares, etc.
- **Example:** A triangle with sides 3, 4, and 5.

- **Use Case Example:** Designing objects and spaces, like creating a triangular garden plot.

## Triangles

- **Definition:** A polygon with three edges and three vertices.
- **Example:** An equilateral triangle with all sides equal.
- **Use Case Example:** Solving problems in trigonometry and surveying land.

## Quadrilaterals

- **Definition:** A polygon with four edges (sides) and four vertices (corners).
- **Example:** A square, with all sides and angles equal.
- **Use Case Example:** Designing floor plans for buildings.

## Coordinate Plane

- **Definition:** A two-dimensional plane defined by a horizontal axis (x-axis) and a vertical axis (y-axis).
- **Example:** Plotting the point (3, 4) on a graph.
- **Use Case Example:** Mapping locations on a coordinate grid in geography.

## Area and Perimeter

- **Definition:** Area is the measure of the space inside a shape; perimeter is the distance around a shape.
- **Example:** The area of a rectangle:  $A = l \times w$   $A = l \times w$ ; the perimeter:  $P = 2(l + w)$   $P = 2(l + w)$   $P = 2(l + w)$ .
- **Use Case Example:** Calculating the amount of paint needed for a wall (area) or the length of fencing for a yard (perimeter).

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## Volume and Surface Area

- **Definition:** Volume is the measure of space inside a 3D object; surface area is the total area of its surface.
- **Example:** The volume of a cube:  $V=a^3$ ; surface area:  $SA=6a^2$ .
- **Use Case Example:** Determining the capacity of a container (volume) or the material needed to cover it (surface area).

## Pythagorean Theorem

- **Definition:** In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
- **Example:** For a triangle with sides 3, 4, and 5,  $3^2+4^2=5^2$ .
- **Use Case Example:** Finding the length of a ladder needed to reach a certain height.

## Transformations

- **Definition:** Operations that move or change a shape without altering its size or orientation.
- **Example:** Translating a shape 5 units to the right.
- **Use Case Example:** Computer graphics, where images are moved or rotated on the screen.

## Congruence

- **Definition:** Two shapes are congruent if they have the same size and shape.
- **Example:** Two triangles with identical sides and angles.

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- **Use Case Example:** Ensuring parts are the same size in manufacturing.

## Similarity

- **Definition:** Two shapes are similar if they have the same shape but not necessarily the same size.
- **Example:** Two rectangles with the same aspect ratio.
- **Use Case Example:** Scaling images in graphic design while maintaining proportions.

## Trigonometry

- **Definition:** The study of relationships between the angles and sides of triangles.
- **Example:** Using sine, cosine, and tangent functions to solve for missing sides.
- **Use Case Example:** Calculating heights of buildings or mountains using angles of elevation.

## Circles

- **Definition:** A round shape with all points equidistant from the center.
- **Example:** The equation of a circle:  $(x-h)^2+(y-k)^2=r^2$
- **Use Case Example:** Designing round objects like wheels or circular plots of land.

## Analytic Geometry

- **Definition:** The study of geometry using a coordinate system and algebraic principles.

- **Example:** Finding the distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  using the distance formula  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .
- **Use Case Example:** Engineering, where precise measurements are needed.

## Geometric Constructions

- **Definition:** Creating geometric figures using only a compass and straightedge.
- **Example:** Constructing the perpendicular bisector of a line segment.
- **Use Case Example:** Drafting and design, where precise constructions are necessary.

## Non-Euclidean Geometry

- **Definition:** Geometry based on a set of axioms that differ from Euclidean geometry, such as hyperbolic and elliptic geometry.
- **Example:** In hyperbolic geometry, the sum of the angles of a triangle is less than 180 degrees.
- **Use Case Example:** Understanding the shape of the universe in cosmology.

## Algebraic Geometry

- **Definition:** The study of geometric properties of solutions to polynomial equations.
- **Example:** The curve defined by the equation  $y^2 = x^3 + ax + b$  (an elliptic curve).
- **Use Case Example:** Cryptography, where algebraic geometry is used to create secure communication protocols.

## Projective Geometry

- **Definition:** The study of geometric properties that are invariant under projection.
- **Example:** Understanding how parallel lines can appear to converge at a point in a perspective drawing.
- **Use Case Example:** Computer graphics and vision, particularly in rendering 3D scenes onto 2D screens.

## Discrete Geometry

- **Definition:** The study of geometric objects and properties that are combinatorial in nature.
- **Example:** Analyzing the properties of polyhedra, graphs, and tilings.
- **Use Case Example:** Network design and analysis.

## Miscellaneous

- **Definition:** Additional geometric topics that don't fit neatly into other categories.
- **Example:** Studying fractals, which exhibit self-similarity at different scales.
- **Use Case Example:** Modeling natural phenomena like coastlines and mountains.

## Trigonometry

Trigonometry is the study of relationships between the angles and sides of triangles. It extends to periodic functions and waves, making it essential in various fields such as physics, engineering, and astronomy.

## Right Triangles & Trigonometry

- **Definition:** Studying the properties of right-angled triangles.

- **Example:** Using the Pythagorean theorem and trigonometric ratios like sine, cosine, and tangent.
- **Use Case Example:** Determining the height of a building using a measured angle and distance.

## Trigonometric Functions

- **Definition:** Functions that relate the angles of a triangle to the lengths of its sides.
- **Example:** Sine ( $\sin$ ), Cosine ( $\cos$ ), and Tangent ( $\tan$ ).
- **Use Case Example:** Modeling periodic phenomena like sound waves.

## Non-right Triangles & Trigonometry

- **Definition:** Studying triangles that are not right-angled, using laws like the Law of Sines and Law of Cosines.
- **Example:** Finding unknown sides and angles using  $a \sin A = b \sin B = c \sin C$  (Law of Sines).
- **Use Case Example:** Navigation and surveying, where triangles are often not right-angled.

## Trigonometric Equations and Identities

- **Definition:** Equations involving trigonometric functions and their properties.
- **Example:**  $\sin^2(x) + \cos^2(x) = 1$  (Pythagorean identity).
- **Use Case Example:** Solving problems in physics and engineering where waveforms are analyzed.

## Topology

Topology is the study of properties of space that are preserved under continuous transformations. It focuses on concepts such as continuity, compactness, and connectedness, often considering spaces that are more abstract than those in classical geometry.

### Point-set Topology

- **Definition:** The study of the more abstract properties of spaces and continuous maps.
- **Example:** Understanding open and closed sets in a topological space.
- **Use Case Example:** Analyzing properties like connectedness and compactness in mathematical spaces.

### Algebraic Topology

- **Definition:** Using tools from abstract algebra to study topological spaces.
- **Example:** Studying the fundamental group, which captures information about the shape of a space.
- **Use Case Example:** Classifying surfaces and understanding more complex structures like knots.

### Differential Topology

- **Definition:** The field dealing with differentiable functions on differentiable manifolds.
- **Example:** Analyzing smooth curves and surfaces.
- **Use Case Example:** Studying the properties of smooth manifolds, important in modern physics.

### Topological Spaces

- **Definition:** The basic object of study in topology, consisting of a set of points along with a set of neighbourhoods for each point.
- **Example:** A simple topological space could be the set of real numbers with the standard topology.
- **Use Case Example:** Studying convergence, continuity, and compactness in various mathematical contexts.

## Fractal Geometry

Fractal geometry studies structures that are self-similar across different scales and can be described by fractal dimensions. These objects appear complex and detailed no matter how much they are magnified.

- **Definition:** The study of complex geometric shapes that can be split into parts, each of which is a reduced-scale copy of the whole.
- **Example:** The Mandelbrot set, a famous fractal.
- **Use Case Example:** Modeling natural phenomena like coastlines, mountain ranges, and snowflakes.

## Differential Geometry

Differential geometry uses the techniques of calculus and linear algebra to study problems in geometry. It focuses on curves, surfaces, and higher-dimensional manifolds, exploring their properties and behaviors in curved spaces.

- **Definition:** The study of curves, surfaces, and shapes using differential and integral calculus.
- **Example:** Understanding the curvature of a surface at a point.
- **Use Case Example:** General relativity, where the curvature of space-time is a central concept.

## Measure Theory

Measure theory is a branch of mathematics that investigates the concept of measure, which generalizes notions such as length, area, and volume. It is fundamental to probability theory and integrates seamlessly with calculus to handle more complex functions and spaces.

- **Definition:** The study of measures, which generalize notions of length, area, and volume.
- **Example:** Defining the Lebesgue measure on the real line.
- **Use Case Example:** Probability theory, where measures are used to define probabilities rigorously.

## 4. Changes

### Calculus

Calculus is the mathematical study of continuous change, dealing with concepts such as limits, derivatives, integrals, and infinite series.

### Differential Calculus

- **Definition:** The branch of calculus concerned with the study of rates at which quantities change.
- **Example:** Finding the derivative of  $f(x) = x^2$  gives  $f'(x) = 2x$ .
- **Use Case Example:** Calculating the velocity of an object at a specific time if the position function is known.

### Limits and Continuity

- **Definition:** Limits describe the behavior of a function as its input approaches a certain value. Continuity ensures that small changes in input result in small changes in output.
- **Example:** The limit of  $f(x) = \sin(x)$  as  $x$  approaches 0 is 0.
- **Use Case Example:** Determining the behavior of functions at points where they aren't explicitly defined.

### Derivatives: Definition and Basic Rules

- **Definition:** A derivative represents the rate of change of a function with respect to a variable. Basic rules include the power rule, product rule, quotient rule, and chain rule.
- **Example:** Using the power rule, the derivative of  $x^3$  is  $3x^2$ .

- **Use Case Example:** Analyzing the slope of a curve at any given point.

## Derivatives: Chain Rule and Other Advanced Topics

- **Definition:** The chain rule is used to differentiate composite functions. Advanced topics include implicit differentiation, higher-order derivatives, and related rates.
- **Example:** If  $y = \sin(3x)$ , using the chain rule,  $y' = 3\cos(3x)$ .
- **Use Case Example:** Solving problems involving related rates, like the rate at which the area of a balloon changes as it inflates.

## Applications of Derivatives

- **Definition:** Using derivatives to solve practical problems such as finding maximum and minimum values, and solving optimization problems.
- **Example:** Finding the maximum profit by setting the derivative of the profit function to zero and solving for the variable.
- **Use Case Example:** Determining the optimal dimensions of a box to maximize volume while minimizing material cost.

## Analyzing Functions

- **Definition:** Using derivatives to study the properties of functions, such as intervals of increase and decrease, concavity, and inflection points.
- **Example:** Determining where a function is increasing or decreasing by analyzing the sign of its derivative.
- **Use Case Example:** Graphing functions to understand their behavior over different intervals.

## Parametric Equations, Polar Coordinates, and Vector-valued Functions

- **Definition:** Parametric equations express a set of related quantities as explicit functions of an independent parameter. Polar coordinates represent points in a plane using a radius and angle. Vector-valued functions assign vectors to each point in a domain.
- **Example:** Parametric equations:  $x = \cos(t)$ ,  $y = \sin(t)$  for  $t \in [0, 2\pi]$ ; Polar coordinates:  $r = 2$ ,  $\theta = \frac{\pi}{4}$ ; Vector-valued function:  $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ .
- **Use Case Example:** Describing the trajectory of a projectile using parametric equations.

**Integral Calculus** Integral calculus is the branch of calculus that deals with integrals and their properties.

### Integrals

- **Definition:** Integrals represent the accumulation of quantities and can be thought of as the area under a curve.
- **Example:** The integral of  $f(x) = x^2$  from 0 to 2 is  $\int_0^2 x^2 dx = \frac{8}{3}$ .
- **Use Case Example:** Calculating the total distance traveled by an object given its velocity over time.

### Differential Equations

- **Definition:** Equations that involve derivatives and describe how quantities change.
- **Example:** The differential equation  $\frac{dy}{dx} = y$  has the solution  $y = Ce^x$ .
- **Use Case Example:** Modeling population growth, where the rate of change of the population is proportional to the current population.

## Applications of Integrals

- **Definition:** Using integrals to solve practical problems such as finding areas, volumes, and accumulated quantities.
- **Example:** Finding the volume of a solid of revolution by integrating the area of cross-sections.
- **Use Case Example:** Determining the amount of work done by a force over a distance.

## Parametric Equations, Polar Coordinates, and Vector-valued Functions

- **Definition:** Integrating functions expressed in parametric form, in polar coordinates, or as vector-valued functions.
- **Example:** Finding the length of a curve described by parametric equations:  $x=t^2$ ,  $y=t^3$ .
- **Use Case Example:** Calculating the path length of a curve in physics or engineering.

## Series

- **Definition:** A series is the sum of the terms of a sequence. It can be finite or infinite.
- **Example:** The geometric series  $\sum_{n=0}^{\infty} ar^n$  converges to  $\frac{a}{1-r}$  for  $|r| < 1$ .
- **Use Case Example:** Calculating interest in finance using geometric series.

## Vector Calculus

Vector calculus extends calculus to functions of several variables.

- **Definition:** The study of multivariable functions and their derivatives and integrals.
- **Example:** The gradient of a function  $f(x,y)$  is  $\nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$ .
- **Use Case Example:** Analyzing fluid flow and electromagnetism.

## Dynamical Systems

Dynamical systems study the behavior of systems that evolve over time according to specific rules.

- **Definition:** Systems that describe the time-dependent evolution of points in a given space.
- **Example:** The logistic map  $x_{n+1} = rx_n(1-x_n)$  models population dynamics.
- **Use Case Example:** Predicting weather patterns and modeling the spread of diseases.

## Chaos Theory

Chaos theory deals with systems that are highly sensitive to initial conditions.

- **Definition:** The study of unpredictable and complex dynamic systems.
- **Example:** The Lorenz attractor, which models atmospheric convection and exhibits chaotic behavior.
- **Use Case Example:** Explaining and predicting phenomena in fields like meteorology, engineering, and economics.

## Complex Analysis

Complex analysis studies functions of complex variables.

- **Definition:** The study of functions that operate on complex numbers.
- **Example:** The function  $f(z) = e^z$  where  $z$  is a complex number.
- **Use Case Example:** Electrical engineering, where complex numbers are used to analyze AC circuits.

# Resources

## 1. Number Systems

### 1. Natural Numbers (N), Whole Numbers (W), Integers (Z), Rational Numbers (Q), Real Numbers (R), Complex Numbers (C), Algebraic Numbers, Transcendental Numbers:

- **Books:**
  - "Elementary Number Theory" by David M. Burton
  - "A Classical Introduction to Modern Number Theory" by Kenneth Ireland and Michael Rosen
- **Courses:**
  - MIT OpenCourseWare: Course on Number Theory
- **Online Resources:**
  - Khan Academy: [Number Theory](#)
  - Coursera: [Introduction to Number Theory](#)

### 2. Binary Numbers, Octal Numbers, Decimal Numbers, Hexadecimal Numbers:

- **Books:**
  - "Discrete Mathematics and Its Applications" by Kenneth H. Rosen
- **Online Resources:**
  - Khan Academy: [Binary and Other Base Systems](#)
  - Brilliant.org: Number Systems

### 3. p-adic Numbers:

- **Books:**
  - "p-adic Numbers: An Introduction" by Fernando Q. Gouvêa
- **Courses:**
  - MIT OpenCourseWare: Advanced Number Theory

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## 2. Structures

### 1. Algebra, Foundations, Algebraic expressions, Linear equations and inequalities, Graphing lines and slope:

- **Books:**
  - "Algebra" by Michael Artin
  - "Introduction to Algebra" by Peter J. Cameron
- **Courses:**
  - Khan Academy: [Algebra](#)
  - MIT OpenCourseWare: Linear Algebra

### 2. Abstract Algebra: Groups, Rings, Fields, Modules, Algebras, Galois Theory, Boolean Algebra:

- **Books:**
  - "Abstract Algebra" by David S. Dummit and Richard M. Foote
  - "A Book of Abstract Algebra" by Charles C. Pinter
- **Courses:**
  - MIT OpenCourseWare: Abstract Algebra
  - Khan Academy: [Abstract Algebra](#)

### 3. Linear Algebra: Vectors and spaces, Matrix transformations:

- **Books:**
  - "Linear Algebra Done Right" by Sheldon Axler
  - "Introduction to Linear Algebra" by Gilbert Strang
- **Courses:**
  - MIT OpenCourseWare: Linear Algebra
  - Khan Academy: [Linear Algebra](#)

### 4. Number Theory: Divisibility, Prime numbers, Diophantine equations, Modular arithmetic, Cryptography:

- **Books:**
  - "An Introduction to the Theory of Numbers" by G.H. Hardy and E.M. Wright
- **Courses:**
  - MIT OpenCourseWare: Number Theory

- Coursera: [Cryptography](#)

## 5. Combinatorics: Permutations, Combinations, Graph Theory, Ramsey Theory:

- **Books:**
  - "Introduction to Graph Theory" by Douglas B. West
  - "Combinatorial Problems and Exercises" by László Lovász
- **Courses:**
  - MIT OpenCourseWare: Combinatorics
  - Khan Academy: [Combinatorics](#)

## 6. Order Theory: Partially ordered sets, Lattices:

- **Books:**
  - "Introduction to Lattices and Order" by B. A. Davey and H. A. Priestley
- **Courses:**
  - MIT OpenCourseWare: Algebra and Lattices

## 7. Logic: Propositional logic, Predicate logic, Set theory, Model theory, Proof theory, Computability theory:

- **Books:**
  - "Introduction to Logic" by Patrick Suppes
  - "Set Theory and Logic" by Robert R. Stoll
- **Courses:**
  - Coursera: [Introduction to Logic](#)
  - MIT OpenCourseWare: Set Theory

## 8. Information Theory: Entropy, Coding theory:

- **Books:**
  - "Elements of Information Theory" by Thomas M. Cover and Joy A. Thomas
- **Courses:**
  - MIT OpenCourseWare: Information Theory

## 3. Spaces

## 1. Geometry: Lines, Angles, Shapes, Triangles, Quadrilaterals, Coordinate plane:

### ○ Books:

- "Geometry" by David A. Brannan, Matthew F. Esplen, and Jeremy J. Gray

### ○ Courses:

- Khan Academy: [Geometry](#)

## 2. Trigonometry: Right triangles & trigonometry, Trigonometric functions:

### ○ Books:

- "Trigonometry" by I.M. Gelfand and Mark Saul

### ○ Courses:

- Khan Academy: [Trigonometry](#)

## 3. Topology: Point-set topology, Algebraic topology, Differential topology:

### ○ Books:

- "Topology" by James R. Munkres
- "Algebraic Topology" by Allen Hatcher

### ○ Courses:

- MIT OpenCourseWare: Topology

## 4. Fractal Geometry, Differential Geometry:

### ○ Books:

- "Fractal Geometry" by Kenneth Falconer
- "Introduction to Differential Geometry" by John M. Lee

### ○ Courses:

- MIT OpenCourseWare: Fractals
- Coursera: [Introduction to Differential Geometry](#)

## 5. Measure Theory:

### ○ Books:

- "Measure Theory" by Paul R. Halmos

### ○ Courses:

- MIT OpenCourseWare: Measure Theory

## 4. Changes

## 1. Calculus: Differential Calculus, Integral Calculus:

- **Books:**
  - "Calculus" by Michael Spivak
  - "Calculus: Early Transcendentals" by James Stewart
- **Courses:**
  - Khan Academy: [Calculus](#)
  - MIT OpenCourseWare: Calculus

## 2. Differential Equations: Ordinary Differential Equations (ODEs), Partial Differential Equations (PDEs):

- **Books:**
  - "Elementary Differential Equations" by William E. Boyce and Richard C. DiPrima
  - "Partial Differential Equations" by Lawrence C. Evans
- **Courses:**
  - MIT OpenCourseWare: Differential Equations

## 3. Dynamical Systems:

- **Books:**
  - "Nonlinear Dynamics and Chaos" by Steven H. Strogatz
- **Courses:**
  - MIT OpenCourseWare: Dynamical Systems

## 4. Complex Analysis:

- **Books:**
  - "Complex Analysis" by Lars Ahlfors
- **Courses:**
  - MIT OpenCourseWare: Complex Variables with Applications

## 5. Functional Analysis:

- **Books:**
  - "Introductory Functional Analysis with Applications" by Erwin Kreyszig
- **Courses:**
  - MIT OpenCourseWare: Functional Analysis

## 6. Fourier Analysis:

- **Books:**
  - "Fourier Analysis: An Introduction" by Elias M. Stein and Rami Shakarchi
- **Courses:**
  - MIT OpenCourseWare: Fourier Analysis

## 7. Numerical Analysis:

- **Books:**
  - "Numerical Analysis" by Richard L. Burden and J. Douglas Faires
- **Courses:**
  - MIT OpenCourseWare: Numerical Analysis

## 8. Probability and Statistics:

- **Books:**
  - "Introduction to Probability" by Dimitri P. Bertsekas and John N. Tsitsiklis
  - "Probability and Statistics" by Morris H. DeGroot and Mark J. Schervish
- **Courses:**
  - Khan Academy: [Probability and Statistics](#)
  - MIT OpenCourseWare: Introduction to Probability and Statistics

## 9. Mathematical Physics: Quantum mechanics, General relativity, Statistical mechanics:

- **Books:**
  - "Mathematical Methods for Physics and Engineering" by K.F. Riley, M.P. Hobson, and S.J. Bence
  - "Quantum Mechanics and Path Integrals" by Richard P. Feynman and Albert R. Hibbs
- **Courses:**
  - MIT OpenCourseWare: Quantum Mechanics
  - MIT OpenCourseWare: General Relativity

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10. **Computational Mathematics: Algorithm theory, Computational complexity, Computer algebra systems:**

- **Books:**
  - "Introduction to Algorithms" by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein
- **Courses:**
  - MIT OpenCourseWare: Introduction to Algorithms

### **Learning Resources**

- **Khan Academy:** [Mathematics](#)
- **Coursera:** [Mathematics Courses](#)
- **edX:** Mathematics Courses
- **MIT OpenCourseWare:** Mathematics Courses
- **Paul's Online Math Notes:** Tutorials
- **Brilliant.org:** Mathematics
- **Wolfram Alpha:** [Mathematics Tools](#)
- **3Blue1Brown (YouTube):** [Channel](#)
- **PatrickJMT (YouTube):** [Channel](#)
- **Just Math Tutorials (YouTube):** [Channel](#)
- **Desmos:** Graphing Calculator
- **Project Euler:** [Mathematical Problems](#)