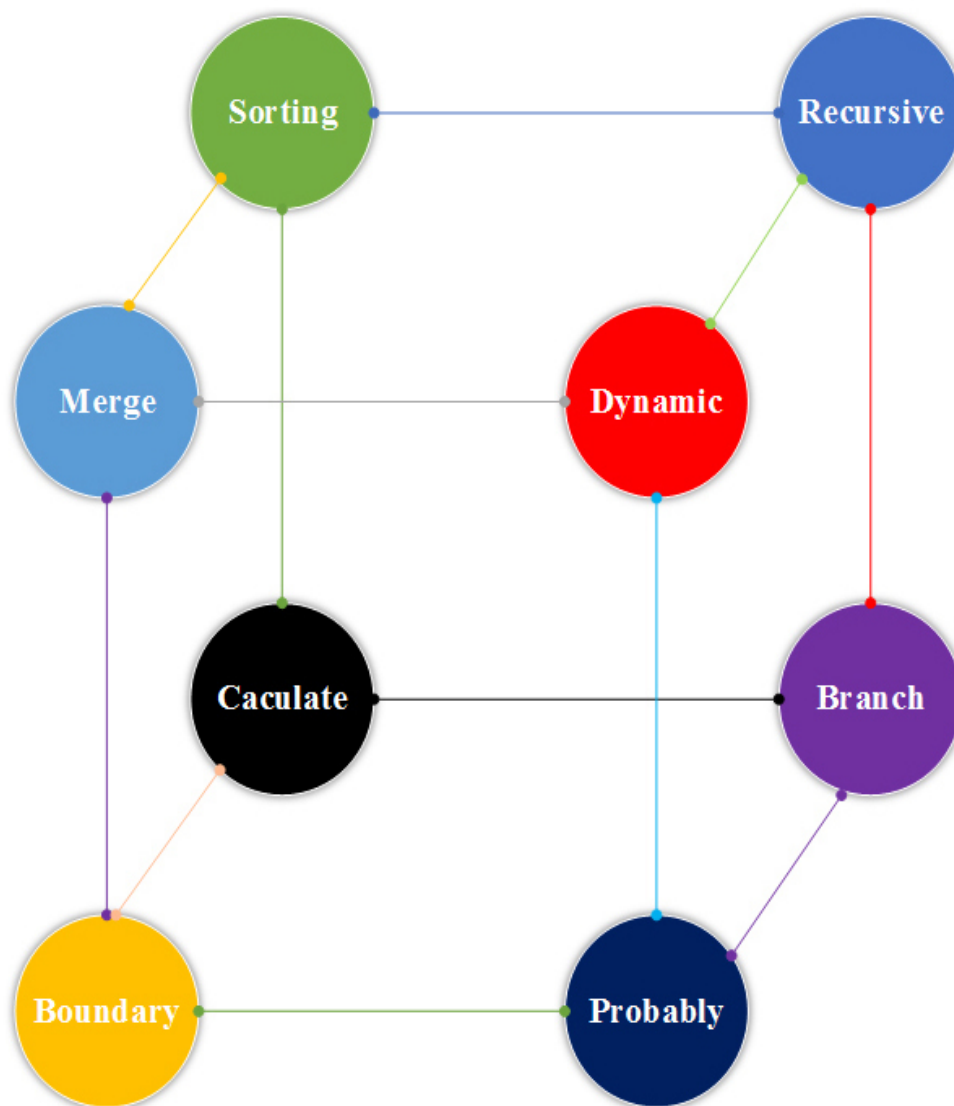
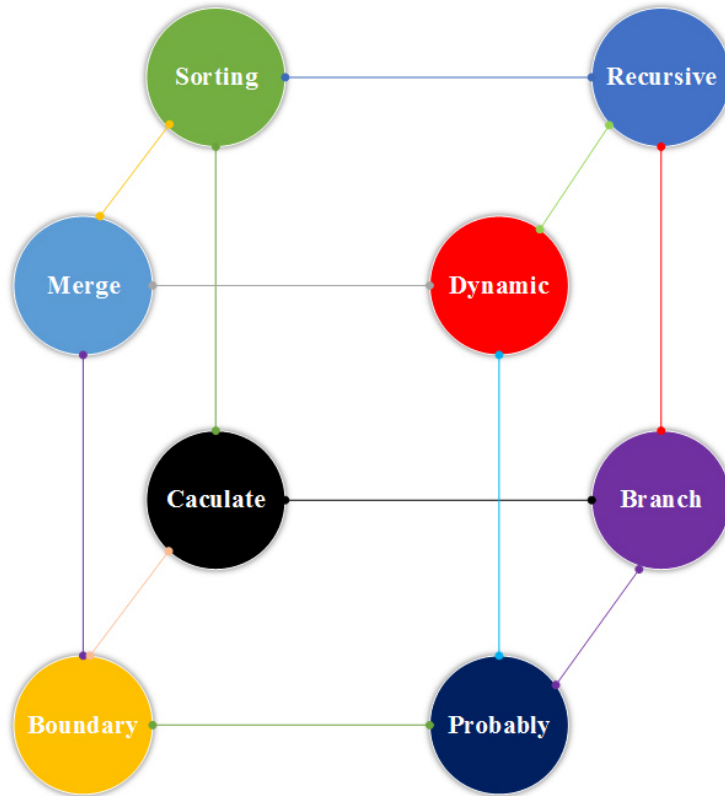


Algorithms Python



Explain Algorithms With Beautiful Pictures

Algorithms Python



Explain Algorithms With Beautiful Pictures

Algorithms Python



YANG HU

Simple is the beginning of wisdom. From the essence of practice, this book to briefly explain the concept, and vividly cultivate programming interest , you will learn it easy fast and well.

<http://en.verejava.com>

Copyright © 2020 Yang Hu

All rights reserved.

ISBN : 9798663846998

CONTENTS

1. [Linear Table Definition](#)
2. [Maximum Value](#)
3. [Bubble Sorting Algorithm](#)
4. [Minimum Value](#)
5. [Select Sorting Algorithm](#)
6. [Linear Table Append](#)
7. [Linear Table Insert](#)

8. [Linear Table Delete](#)
9. [Insert Sorting Algorithm](#)
10. [Reverse Array](#)
11. [Linear Table Search](#)
12. [Dichotomy Binary Search](#)
13. [Shell Sorting](#)
14. [Unidirectional Linked List](#)
 - 14.1 [Create and Traversal](#)
 - 14.2 [Add Node](#)
 - 14.3 [Insert Node](#)
 - 14.4 [Delete Node](#)
15. [Doubly Linked List](#)
 - 15.1 [Create and Traversal](#)
 - 15.2 [Add Node](#)
 - 15.3 [Insert Node](#)
 - 15.4 [Delete Node](#)
16. [One-way Circular LinkedList](#)
 - 16.1 [Initialization and Traversal](#)
 - 16.2 [Insert Node](#)
 - 16.3 [Delete Node](#)
17. [Two-way Circular LinkedList](#)
 - 17.1 [Initialization and Traversal](#)
 - 17.2 [Insert Node](#)
 - 17.3 [Delete Node](#)
18. [Queue](#)
19. [Stack](#)
20. [Recursive Algorithm](#)
21. [Two-way Merge Algorithm](#)

- 22. [Quick Sort Algorithm](#)
- 23. [Binary Search Tree](#)
 - 23.1 [Construct a binary search tree](#)
 - 23.2 [Binary search tree In-order traversal](#)
 - 23.3 [Binary search tree Pre-order traversal](#)
 - 23.4 [Binary search tree Post-order traversal](#)
 - 23.5 [Binary search tree Maximum and minimum](#)
 - 23.6 [Binary search tree Delete Node](#)
- 24. [Binary Heap Sorting](#)
- 25. [Hash Table](#)
- 26. [Graph](#)
 - 26.1 [Directed Graph and Depth-First Search](#)
 - 26.2 [Directed Graph and Breadth-First Search](#)
 - 26.3 [Directed Graph Topological Sorting](#)
- 27. [Towers of Hanoi](#)
- 28. [Fibonacci](#)
- 29. [Dijkstra](#)
- 30. [Mouse Walking Maze](#)
- 31. [Eight Coins](#)
- 32. [Knapsack Problem](#)
- 33. [Josephus Problem](#)

Linear Table Definition

Linear Table:

Sequence of elements, is a one-dimensional array.

1 . Define a one-dimensional array of student scores

0	1	2	3	4	5
90	70	50	80	60	85

length = 6

test_one_array.py

```
import sys
def main ():
    scores = [ 90 , 70 , 50 , 80 , 60 , 85 ]

    length = len( scores)

    for i in range( 0 , length):
        print ( scores[ i], " , " , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

90, 70, 50, 80, 60, 85

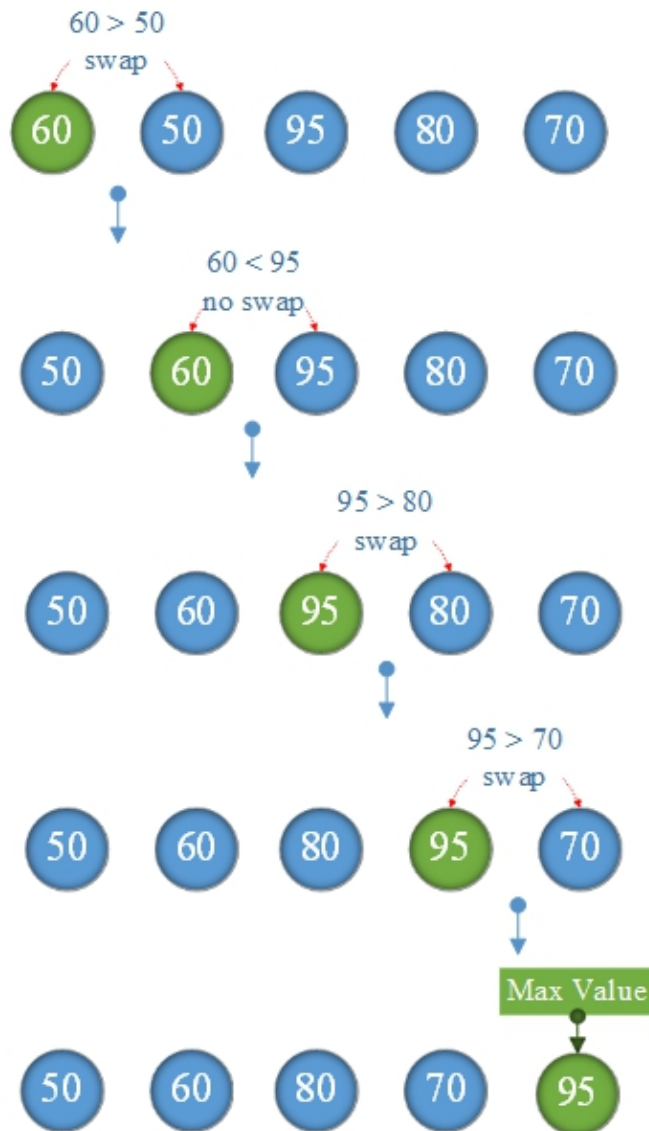
Maximum Value

Maximum of Integer Sequences:



1. Algorithmic ideas

Compare $arrays[i]$ with $arrays[i + 1]$, if $arrays[i] > arrays[i + 1]$ are exchanged. So continue until the last number, $arrays[length - 1]$ is the maximum.



test_max_value.py

```
import sys

def max ( arrays):
    length = len( arrays)
    for i in range( 0 , length- 1 ):
        if ( arrays[ i] > arrays[ i + 1 ]):
            temp = arrays[ i]
            arrays[ i] = arrays[ i + 1 ]
            arrays[ i + 1 ] = temp
    maxValue = arrays[ length - 1 ]
    return maxValue

def main ():
    scores = [ 60 , 50 , 95 , 80 , 70 ]
    maxValue = max( scores)
    print ( "Max Value = " , maxValue)

if __name__ == "__main__" :
    main()
```

Result:

Max Value = 95

Bubble Sorting Algorithm

Bubble Sorting Algorithm:

Compare `arrays[j]` with `arrays[j + 1]` , if `arrays[j] > arrays[j + 1]` are exchanged.

Remaining elements repeat this process, until sorting is completed.

Sort the following numbers from small to large



Explanation:



No sorting,

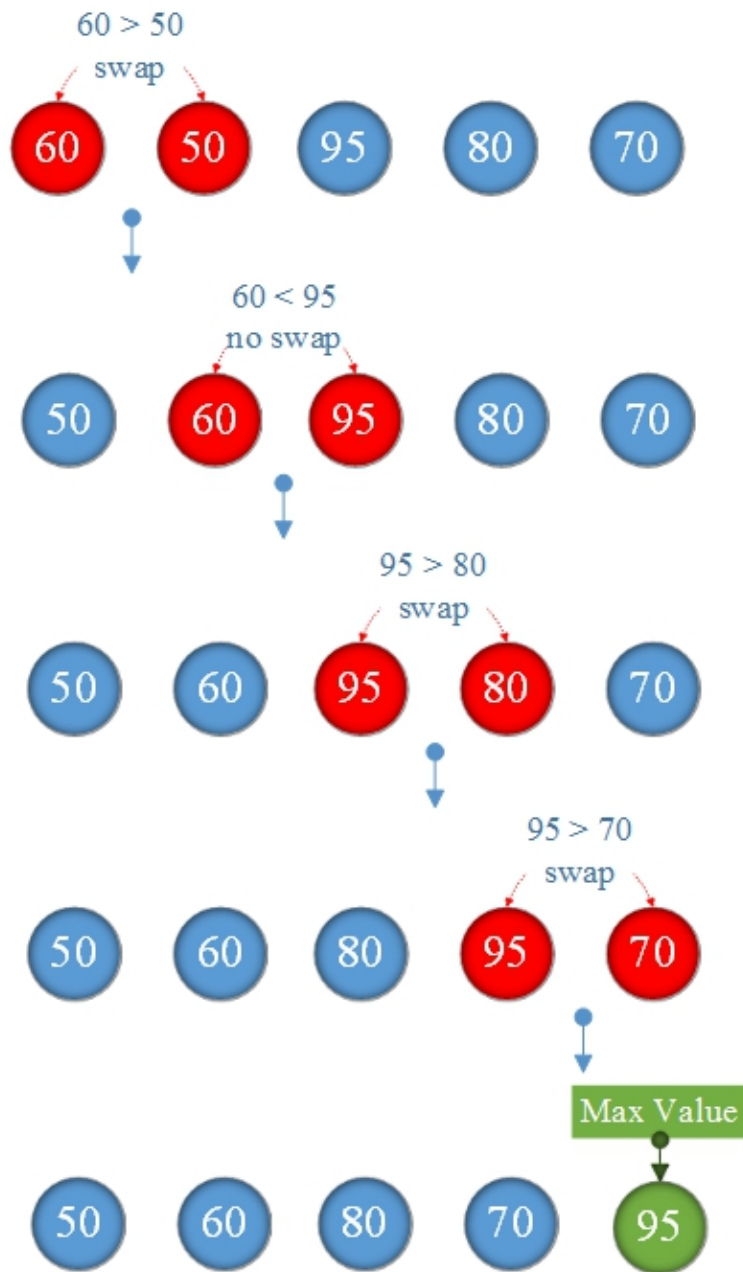


Comparing,

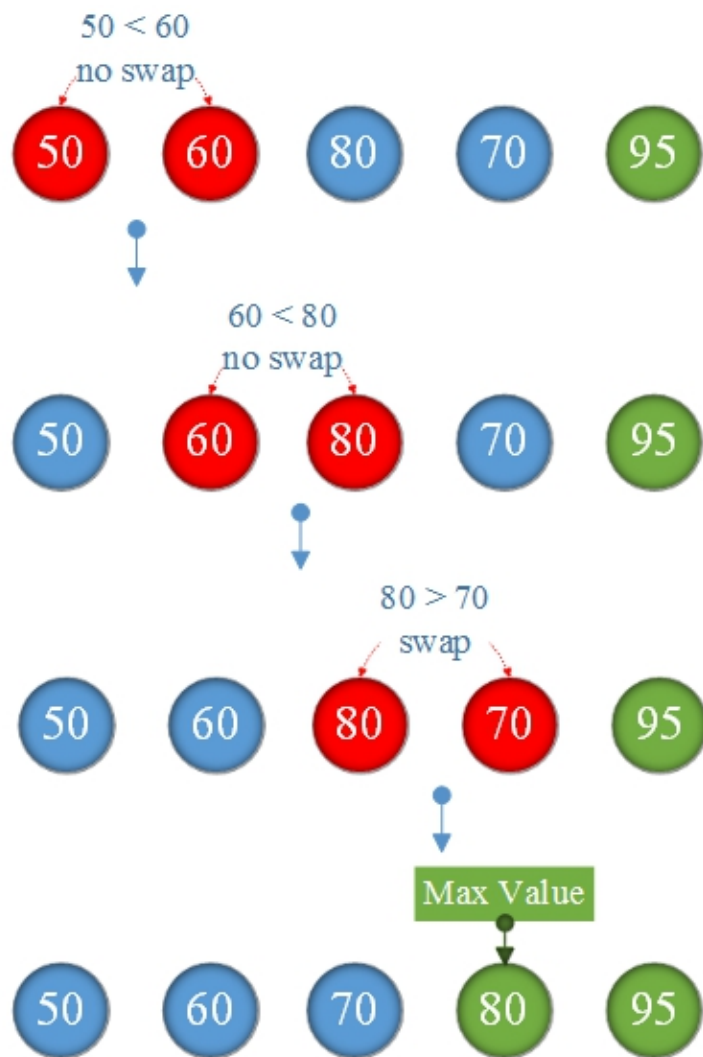


Already sorted

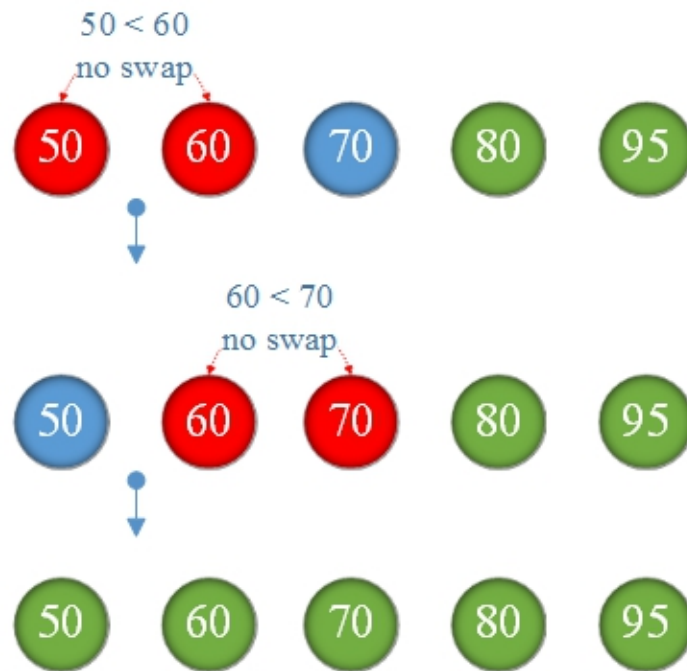
1 . First sorting:



2 . Second sorting:



3 . Third sorting:



No swap so terminate sorting : we can get the sorting numbers from small to large



Create Test File : test_bubble_sort.py

```
import sys

def sort ( arrays):
    length = len( arrays)
    for i in range( 0 , length- 1 ):
        for j in range( 0 , length- i- 1 ):
            if arrays[ j] > arrays[ j+ 1 ]:
                flag = arrays[ j]
                arrays[ j] = arrays[ j+ 1 ]
                arrays[ j+ 1 ] = flag

def main ():
    scores = [ 60 , 50 , 95 , 80 , 70 ]

    sort( scores)

    for score in scores:
        print ( score, ", " , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

50 ,60 ,70 ,80 ,95 ,

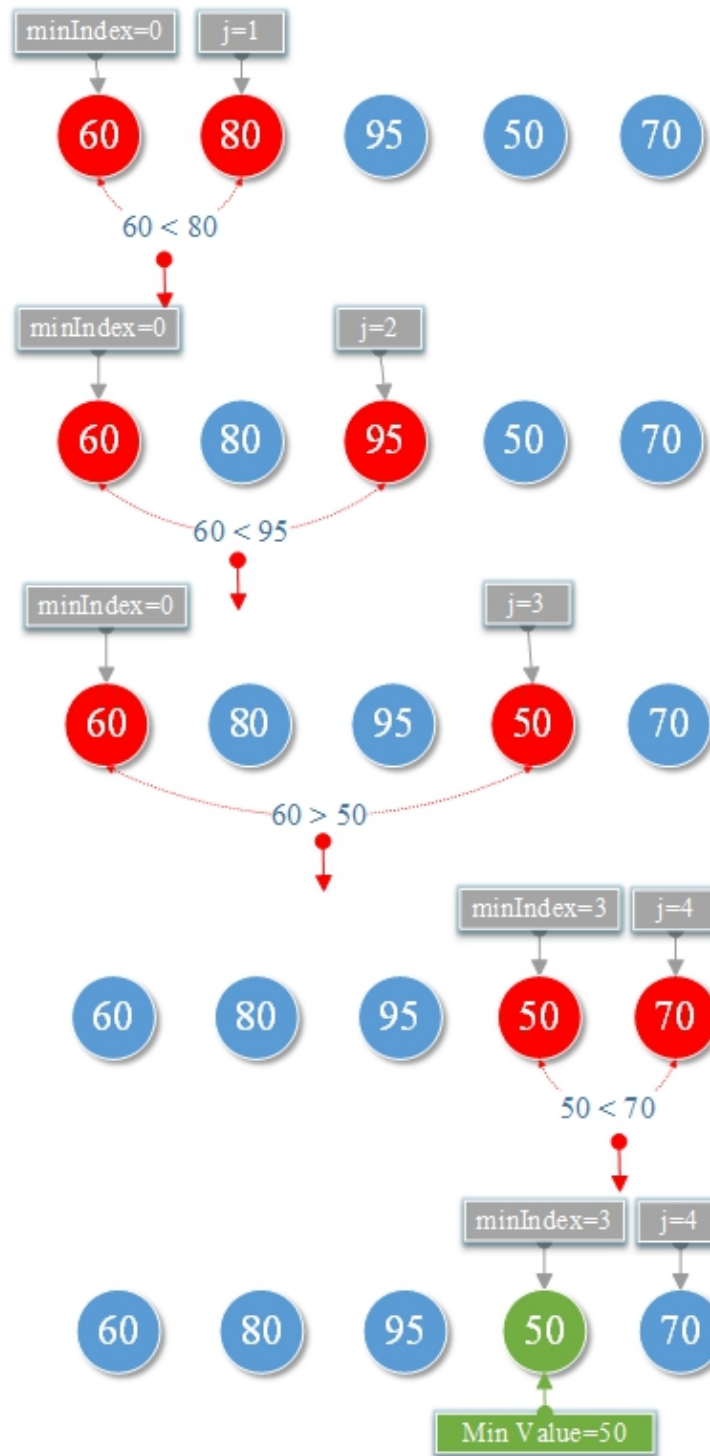
Minimum Value

Search the Minimum of Integer Sequences:



1 . Algorithmic ideas

Initial value `minIndex=0`, `j=1` Compare `arrays[minIndex]` with `arrays[j]`
if `arrays[minIndex] > arrays[j]` then `minIndex=j`, `j++` else `j++`. continue
until the last number, `arrays[minIndex]` is the Min Value.



test_min_value.py

```
import sys

def min ( arrays):
    minIndex = 0 ;
    length = len( arrays) - 1
    for j in range( 0 , length):
        if ( arrays[ minIndex] > arrays[ j]):
            minIndex = j
    return arrays[ minIndex]

def main ():
    scores = [ 60 , 80 , 95 , 50 , 70 ]
    minValue = min( scores)
    print ( "Min Value = " , minValue)

if __name__ == "__main__" :
    main()
```

Result:

Min Value = 50

Select Sorting Algorithm

Select Sorting Algorithm:

Sorts an array by repeatedly finding the minimum element from unsorted part and putting it at the beginning.

Sort the following numbers from small to large



Explanation:



No sorting,

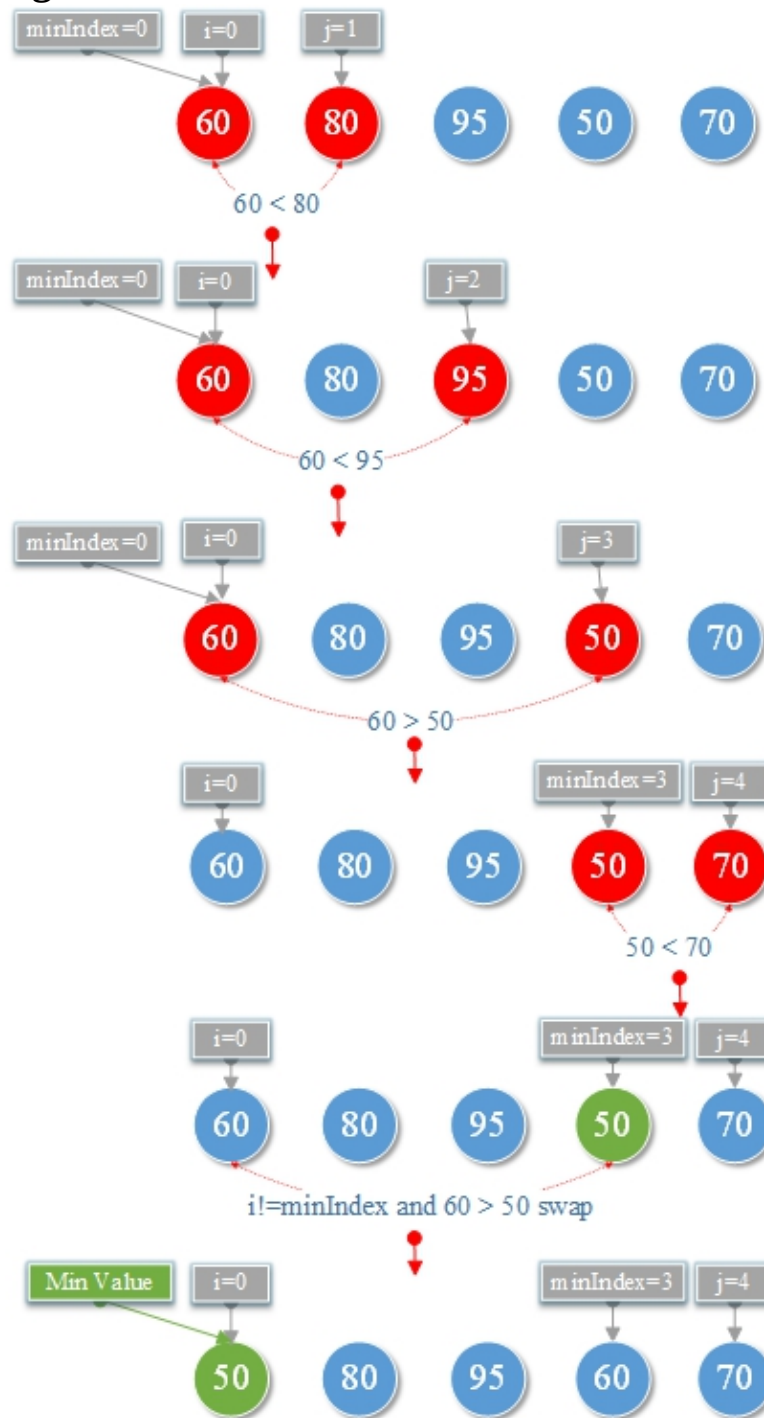


Comparing,

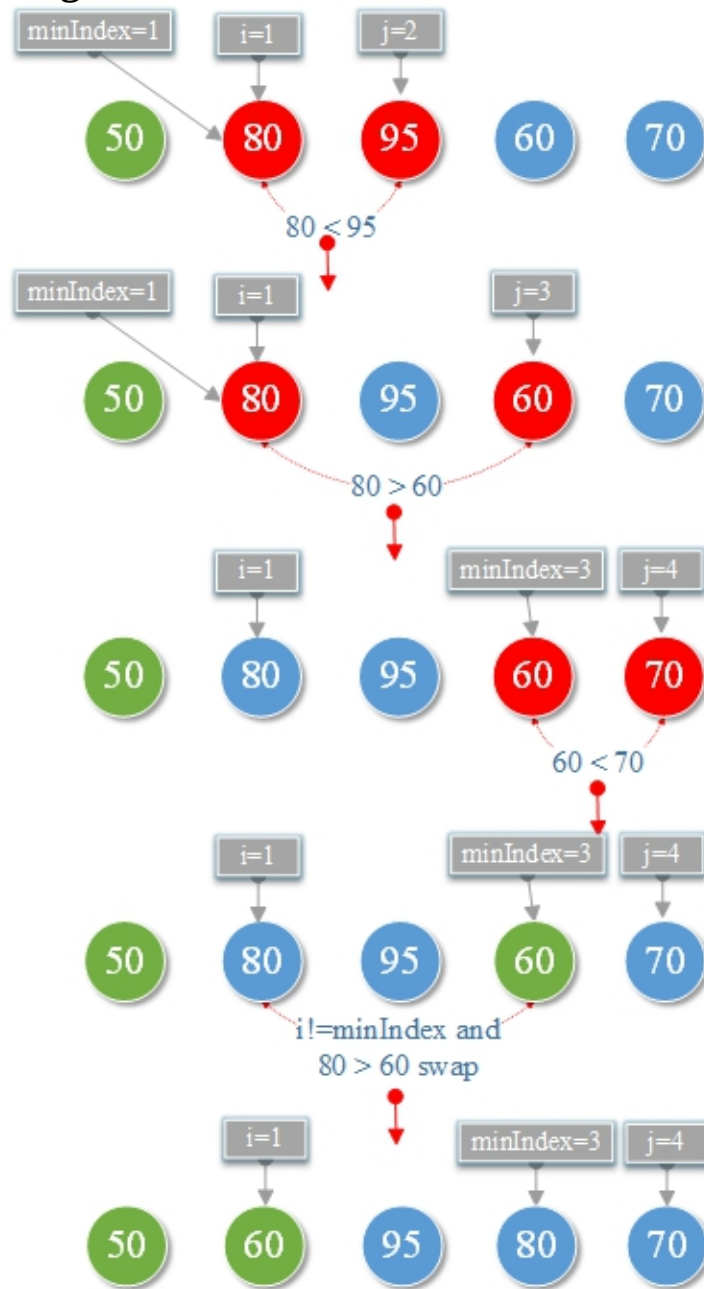


Already sorted.

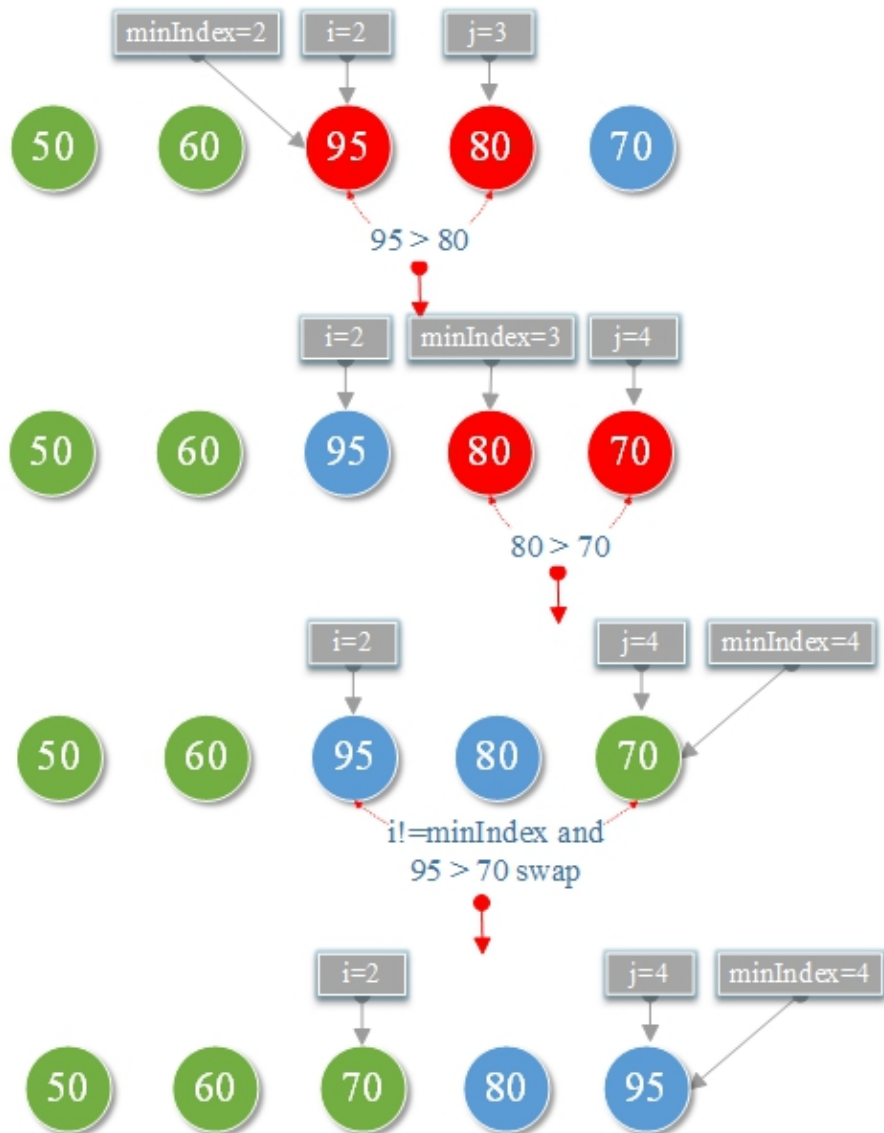
1 . First sorting:



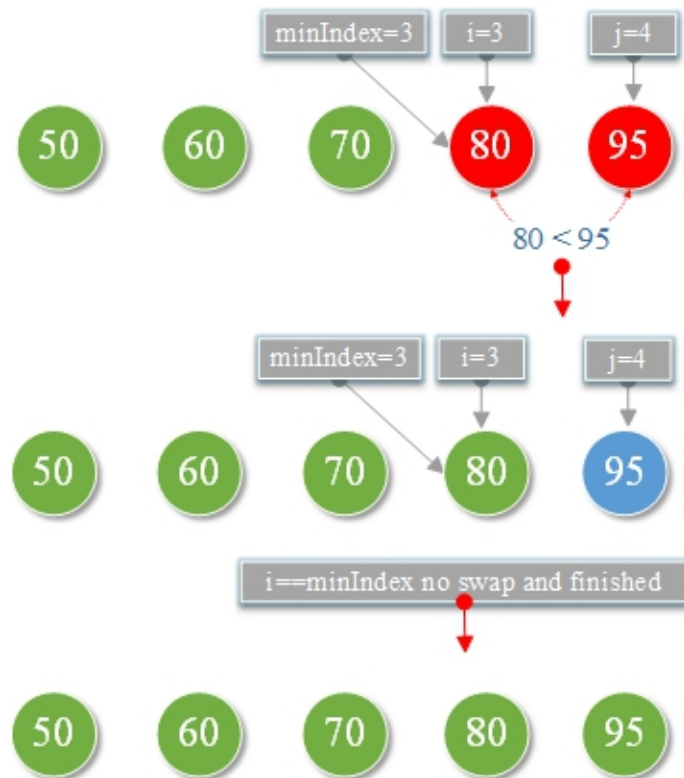
2 . Second sorting:



3 . Third sorting:



4 . Forth sorting:



we can get the sorting numbers from small to large



t est_select_sort.py

```
import sys

def sort ( arrays):
    length = len( arrays) - 1
    min_index = 0 # the index of the selected minimum

    for i in range( 0 , length):
        min_index = i
        # the minimum value of each loop as the first element
        min_value = arrays[ min_index]
        for j in range( i, length):
            # Compare with each element if it is less than the minimum
            value, exchange the min_index
            if min_value > arrays[ j + 1 ]:
                min_value = arrays[ j + 1 ]
                min_index = j + 1

        # if the minimum index changes, the current minimum is
        exchanged with the min_index
        if i != min_index:
            temp = arrays[ i]
            arrays[ i] = arrays[ min_index]
            arrays[ min_index] = temp

def main ():
    scores = [ 60 , 80 , 95 , 50 , 70 ]
    sort( scores)
    for score in scores:
        print ( score, ", " , end= "" )

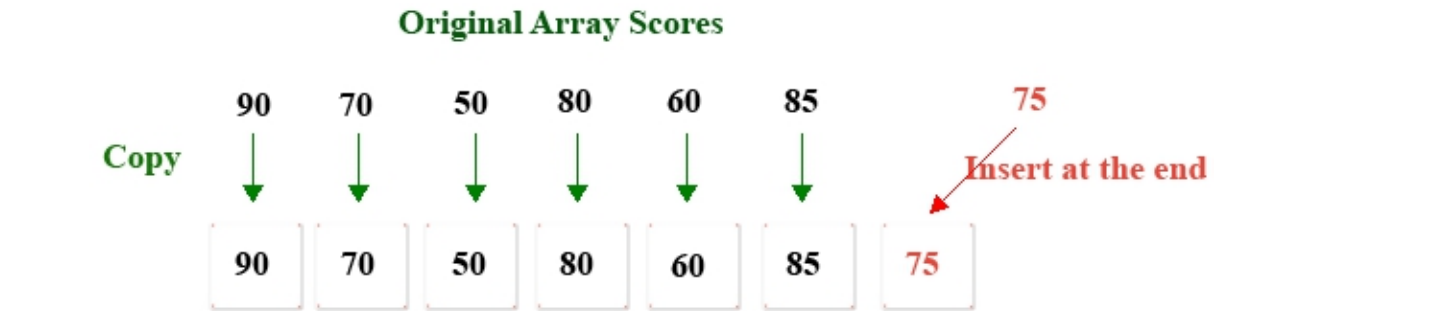
if __name__ == "__main__" :
    main()
```

Result:

50 ,60 ,70 ,80 ,95 ,

Linear Table Append

- 1 . Add a score **75** to the end of the one-dimensional array scores.



Analysis:

1. First create a temporary array(**tempArray**) larger than the original scores array length
2. Copy each value of the scores to **tempArray**
3. Assign 75 to the last index position of **tempArray**
4. Finally assign the **tempArray** pointer reference to the original scores;

test_one_array_append.py

```
import sys

def main():
    scores = [ 90 , 70 , 50 , 80 , 60 , 85 ]
    length = len( scores)

    #create a new temp array, length = length + 1
    tempArray = [ 0 for _ in range( length+ 1 )]

    for i in range( 0 , length):
        tempArray[ i ] = scores[ i]

    tempArray[ length ] = 75
    scores = tempArray

    length = len( scores)
    for i in range( 0 , length):
        print ( scores[ i ] , "," , end= "" )

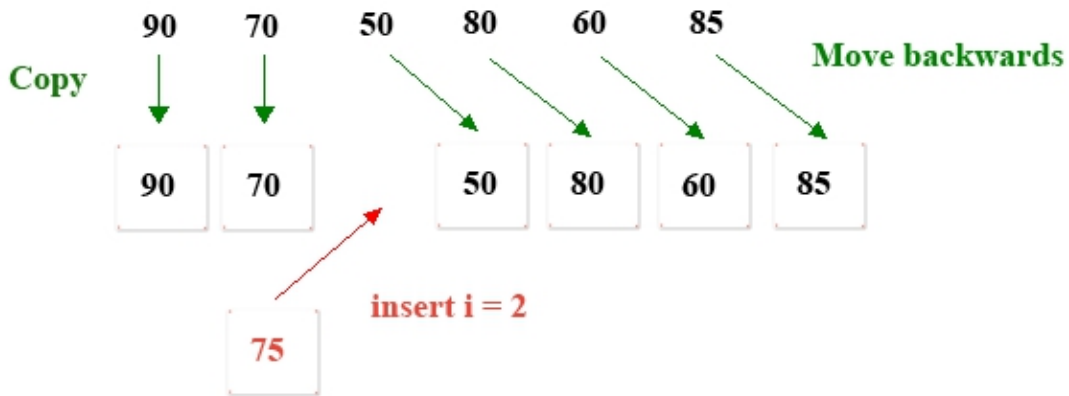
if __name__ == "__main__" :
    main()
```

Result:

90, 70, 50, 80, 60, 85, 75,

Linear Table Insert

1 . Insert a student's score anywhere in the one-dimensional array scores.



Analysis:

1. First create a temporary array **tempArray** larger than the original scores array length
2. Copy each value of the previous value of the scores array from the beginning to the insertion position to **tempArray**
3. Move the scores array from the insertion position to each value of the last element and move it back to **tempArray**
4. Then insert the score 75 to the index of the **tempArray** .
5. Finally assign the **tempArray** pointer reference to the scores;

test_one_array_insert.py

```
import sys

def insert ( array, score, insertIndex):
    length = len( array)
    tempArray = [ 0 for _ in range( length+ 1 )]
    for i in range( 0 , length):
        if i < insertIndex:
            tempArray[ i ] = array[ i]
        else :
            tempArray[ i + 1 ] = array[ i]

    tempArray[ insertIndex] = score
    return tempArray

def main ():
    scores = [ 90 , 70 , 50 , 80 , 60 , 85 ]

    scores = insert( scores, 75 , 2 ) #Insert 75 into the position: index = 2

    length = len( scores)
    for i in range( 0 , length):
        print ( scores[ i] , "," , end= "" )

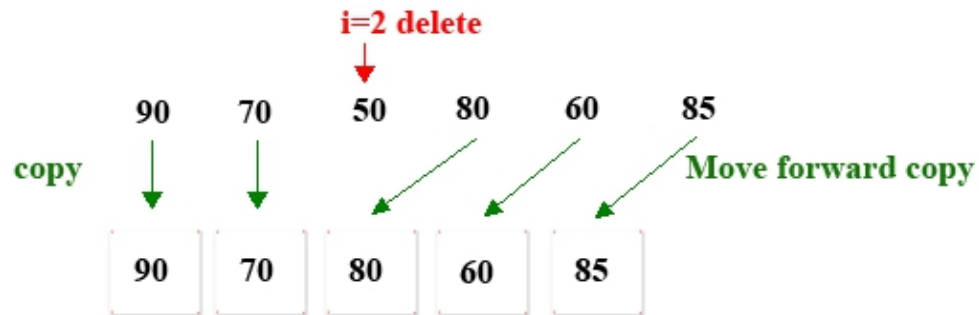
if __name__ == "__main__" :
    main()
```

Result:

90,70,75,50, 80, 60, 85,

Linear Table Delete

1 . Delete the value of the **index=2** from scores array



Analysis:

1. Create a temporary array **tempArray** that length smaller than scores by 1.
2. Copy the data in front of **i=2** to the front of **tempArray**
3. Copy the array after **i=2** to the end of **tempArray**
4. Assign the **tempArray** pointer reference to the scores
5. Printout scores

test_one_array_delete.py

```
import sys

def delete ( array, deleteIndex):
    length = len( array)
    # create a new temp array, length = length - 1
    tempArray = [ 0 for _ in range( length - 1 )]

    for i in range( 0 , length):
        if i < deleteIndex: # Copy the data in front of index to the front of
tempArray
            tempArray[ i] = array[ i]
        if i > deleteIndex: # Copy the array after index to the end of
tempArray
            tempArray[ i - 1 ] = array[ i]

    return tempArray

def main ():
    scores = [ 90 , 70 , 50 , 80 , 60 , 85 ]
    index= int( input( "Please enter the index to be deleted: \n" ))

    scores = delete( scores, index)

    length = len( scores)
    for i in range( 0 , length):
        print ( scores[ i] , "," , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

Please enter the index to be deleted:

2

90 ,70 ,80 ,60 ,85 ,

Insert Sorting Algorithm

Insert Sorting Algorithm:

Take an unsorted new element in the array, compare it with the already sorted element before, if the element is smaller than the sorted element, insert new element to the right position.

Sort the following numbers from small to large



Explanation:



No sorting,

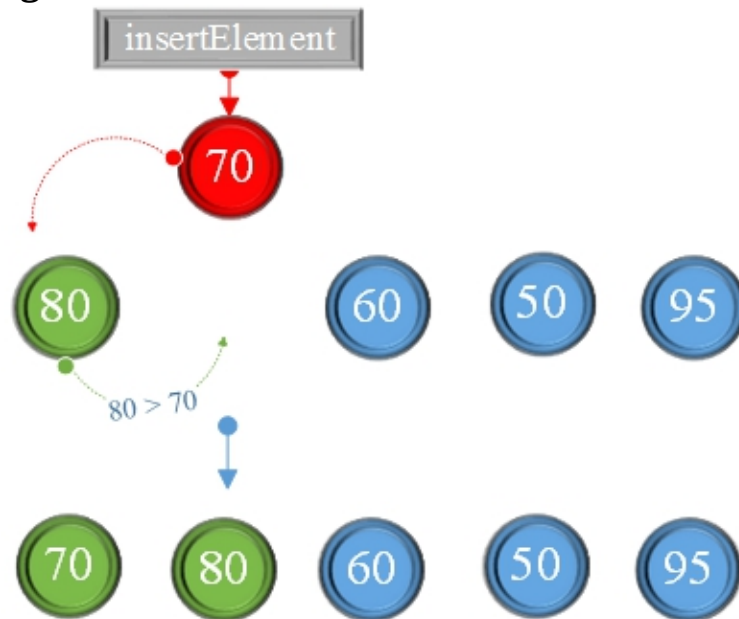


Inserting,

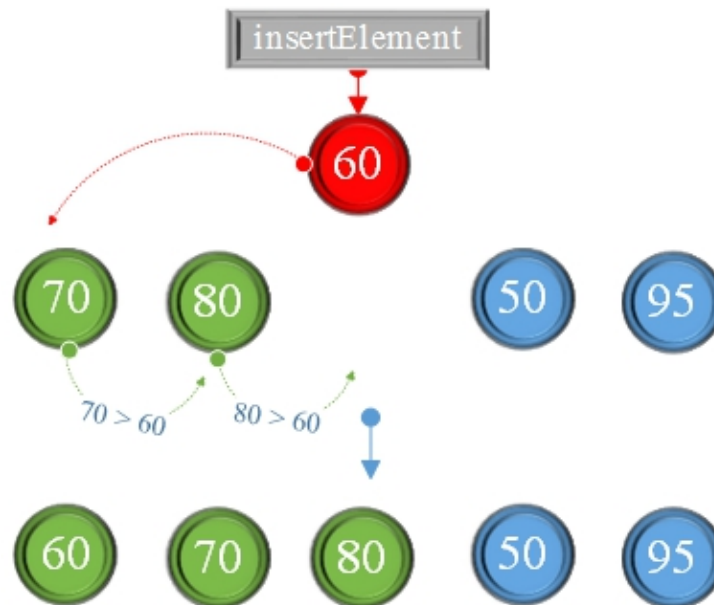


Already sorted

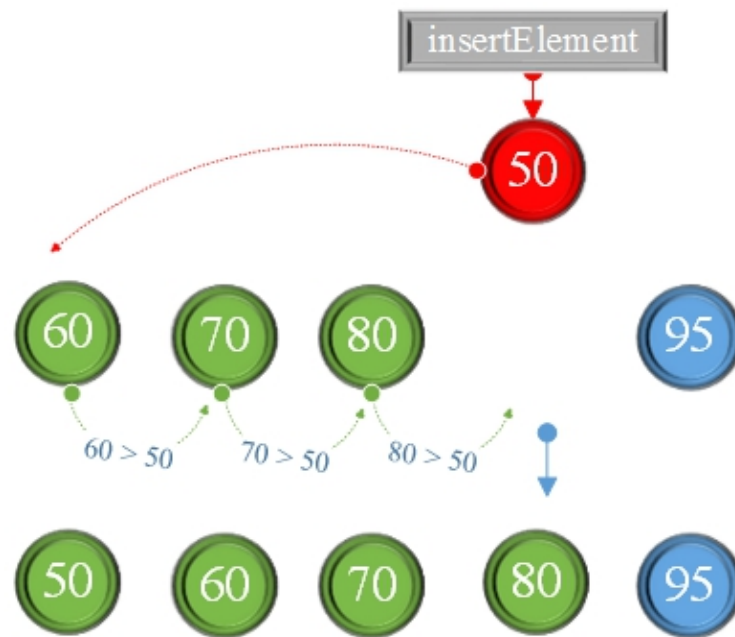
1 . First sorting:



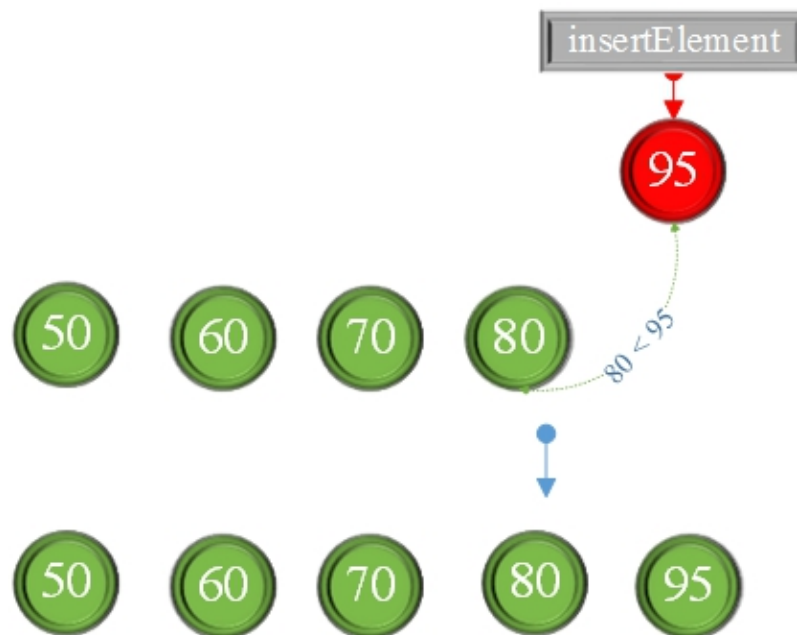
2 . Second sorting:



3 . Third sorting:



4 Third sorting:



test_insert_sort.py

```
import sys

def sort ( arrays):
    length = len( arrays)
    for i in range( 0 , length- 1 ):
        insert_element = arrays[ i] #Take unsorted new elements
        insert_position = i # Inserted position
        for j in range( insert_position - 1 , - 1 , - 1 ):
            # If the new element is smaller than the sorted element, it is
            shifted to the right
            if insert_element < arrays[ j]:
                arrays[ j + 1 ] = arrays[ j]
                insert_position-= 1
        arrays[ insert_position] = insert_element # Insert the new element

def main ():
    scores = [ 80 , 70 , 60 , 50 , 95 ]
    sort( scores)
    for score in scores:
        print ( score, " , " , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

50 ,60 ,70 ,80 ,95 ,

Reverse Array

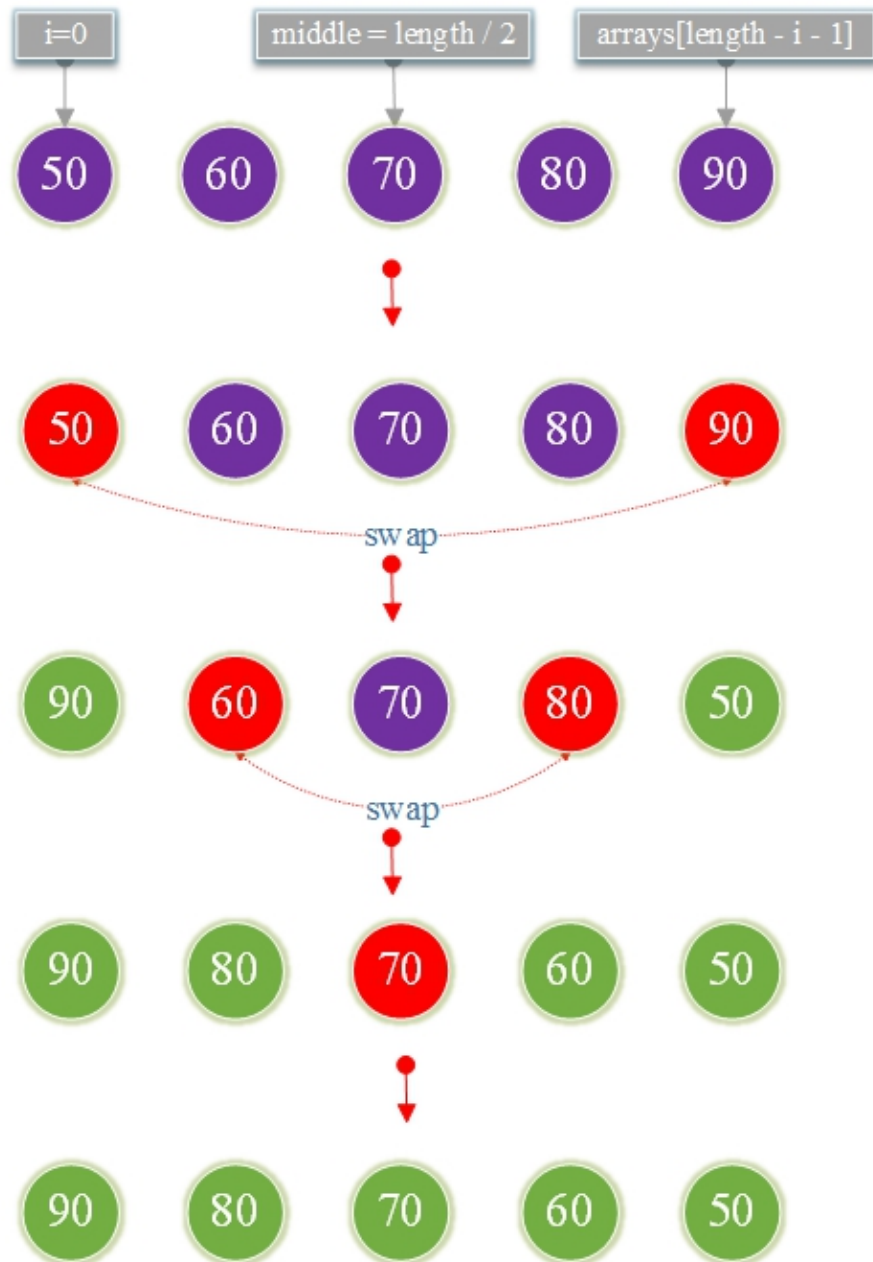
Inversion of ordered sequences:



1 . Algorithmic ideas

Initial $i = 0$ and then swap the first element `arrays[i]` with last element `arrays[length - i - 1]`

Repeat until index of middle $i == \text{length} / 2$.



test_reverse.py

```
import sys

def reverse ( arrays):
    length = len( arrays)
    middle = int( length / 2 )
    for i in range( 0 , middle):
        temp = arrays[ i]
        arrays[ i] = arrays[ length - i -
1 ]
        arrays[ length - i - 1 ] = temp

def main ():
    scores = [ 50 , 60 , 70 , 80 , 90 ]
    reverse( scores)
    for score in scores:
        print ( score, ", " , end= "" )

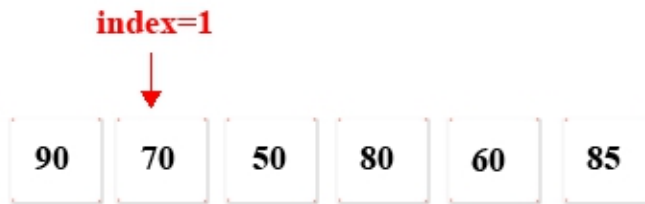
if __name__ == "__main__" :
    main()
```

Result:

90,80,70,60,50,

Linear Table Search

1 . Please enter the value you want to search like : **70** return index.



test_one_array_search.py

```
import sys

def search ( array, value):
    length = len( array)
    for i in range( 0 , length):
        if ( array[ i] == value):
            return i
    return - 1

def main ():
    scores = [ 90 , 70 , 50 , 80 , 60 , 85 ]
    value= int( input( "Please enter the value you want to search : \n" ))

    index = search( scores, value)

    if ( index > 0 ):
        print ( "Found value: " , value , " the index is: " , index)
    else :
        print ( "The value was not found : " , value)

if __name__ == "__main__" :
    main()
```

Result:

Please enter the value you want to search :

70

Found value: 70 the index is: 1

Dichotomy Binary Search

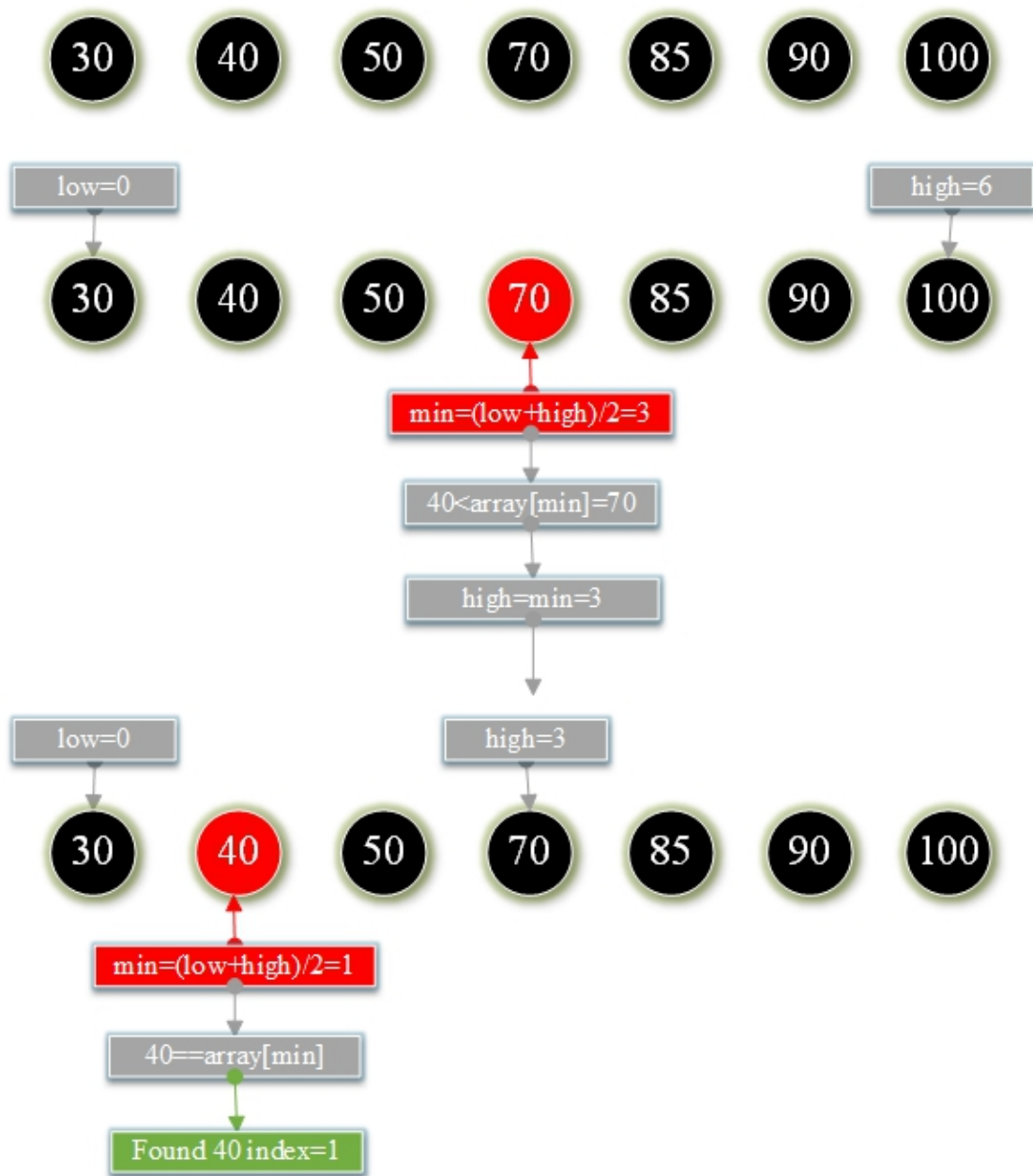
Dichotomy Binary Search:

Find the index position of a given value from an already ordered array.

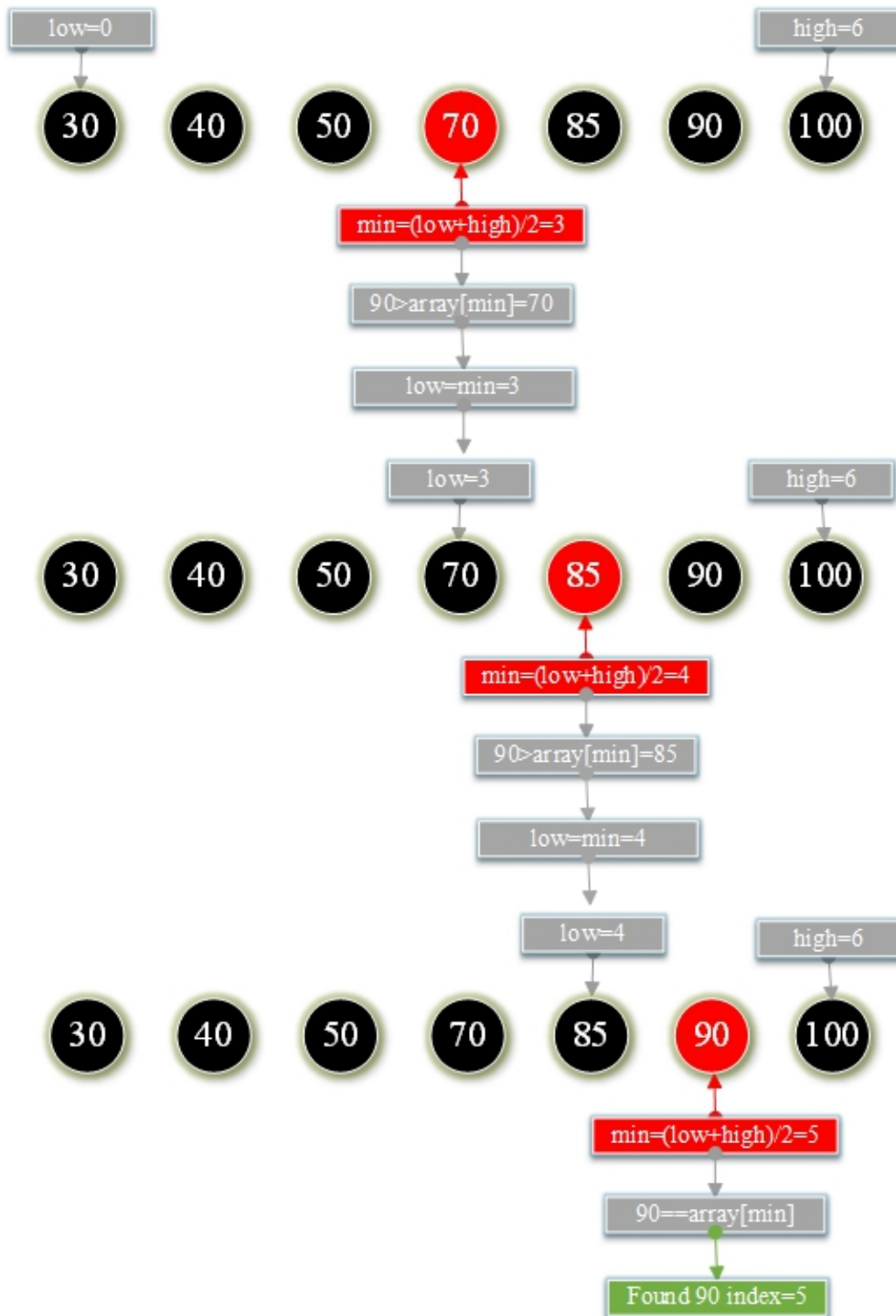


1. Initialize the lowest index `low=0` , the highest index `high=scores.length-1`
2. Find the `searchValue` of the middle index `mid=(low+high)/2` `scores[mid]`
3. Compare the `scores[mid]` with `searchValue`
If the `scores[mid]==searchValue` print current mid index,
If `scores[mid]>searchValue` that the `searchValue` will be found between
`low` and `mid-1`
4. And so on. Repeat step 3 until you find `searchValue` or `low>=high` to terminate the loop.

Example 1 : Find the index of searchValue=40 in the array that has been sorted below.



Example 2 : Find the index of searchValue=90 in the array that has been sorted below.



test_binary_search.py

```
import sys

def binary_search ( arrays, search_value):
    length = len( arrays)
    low = 0
    high = length - 1
    mid = 0
    while low <= high:
        mid = ( low + high) // 2
        if arrays[ mid] == search_value:
            return mid
        elif arrays[ mid] < search_value:
            low = mid + 1
        elif arrays[ mid] > search_value:
            high = mid - 1
    return - 1

def main ():
    scores = [ 30 , 40 , 50 , 70 , 85 , 90 , 100 ]
    search_value = 40 ;
    position = binary_search( scores, search_value)
    print ( search_value, " position:" , position)

    print ( "-----" )

    search_value = 90
    position = binary_search( scores, search_value)
    print ( search_value, " position:" , position)

if __name__ == "__main__" :
    main()
```

Result:

40 position: 1

90 position: 5

Shell Sorting

Shell Sorting:

Shell sort is a highly efficient sorting algorithm and is based on insertion sort algorithm. This algorithm avoids large shifts as in case of insertion sort, if the smaller value is to the far right and has to be moved to the far left.

Sort the following numbers from small to large by Shell Sorting

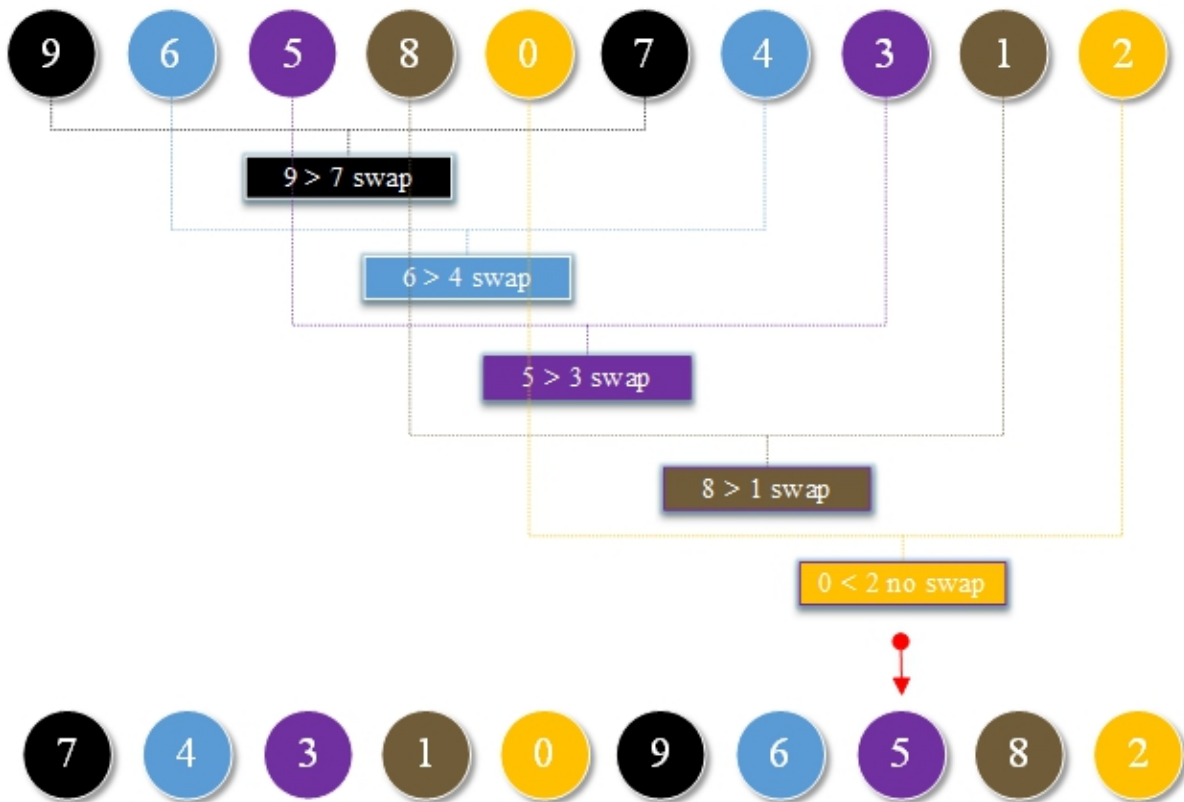


Algorithmic result:

The array is grouped according to a certain increment of subscripts, and the insertion of each group is sorted. As the increment decreases gradually until the increment is 1, the whole data is grouped and sorted.

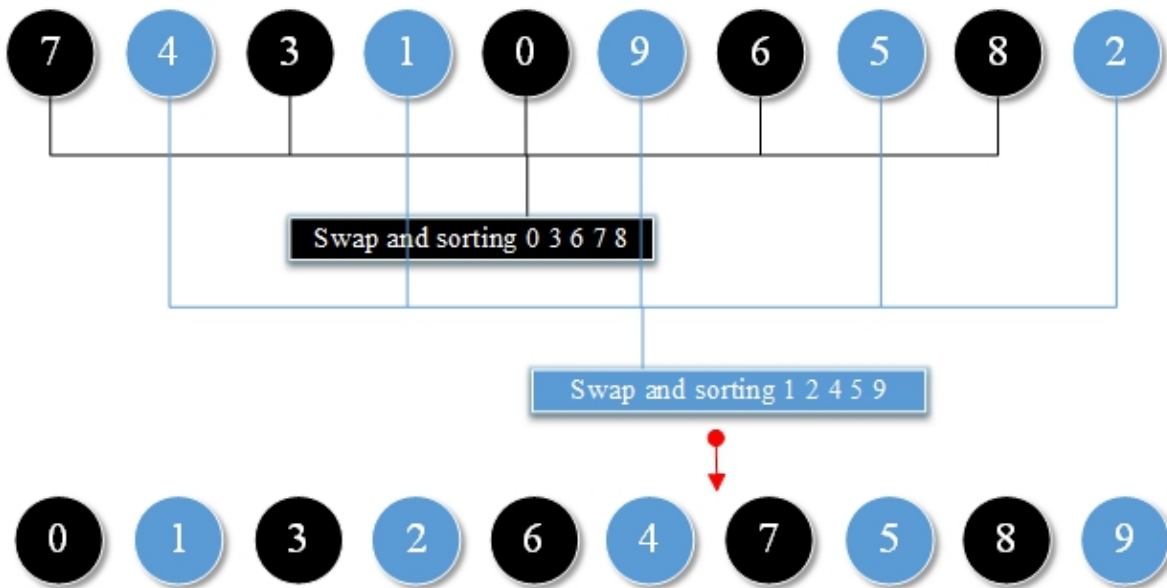
1 . The first sorting :

gap = array.length / 2 = 5



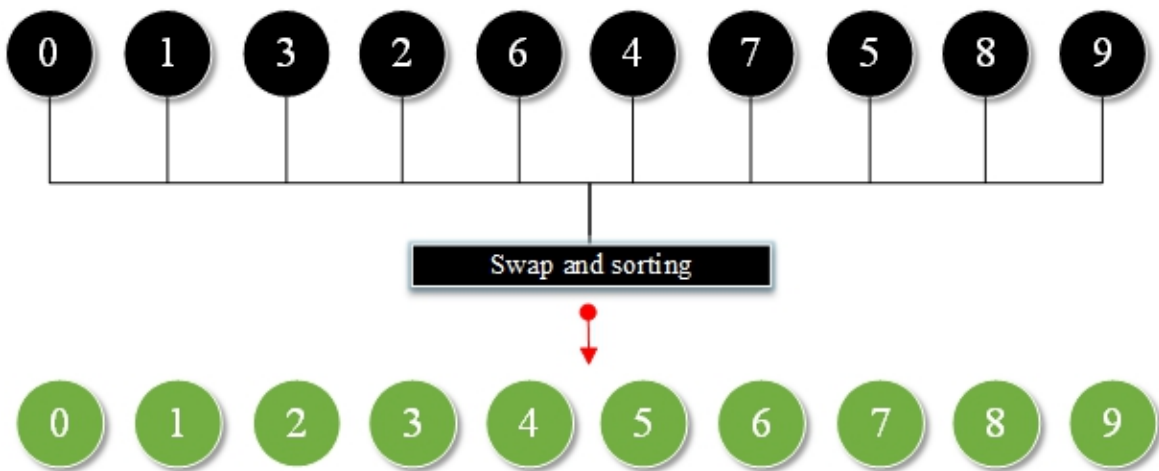
2 . The second sorting :

$$\text{gap} = 5 / 2 = 2$$



3 . The third sorting :

$$\text{gap} = 2 / 2 = 1$$



test_shell_sort.py

```
import sys

def shell_sort ( array):
    length = len( array)
    gap = int( length / 2 )
    while ( gap > 0 ):
        for i in range( gap, length):
            j = i;
            while ( j - gap >= 0 and array[ j] < array[ j
- gap]):
                swap( array, j, j - gap);
                j = j - gap;
            gap = int( gap / 2 )

def swap ( array, a, b):
    array[ a] = array[ a] + array[ b]
    array[ b] = array[ a] - array[ b]
    array[ a] = array[ a] - array[ b]

def main ():
    scores = [ 9 , 6 , 5 , 8 , 0 , 7 , 4 , 3 , 1 , 2 ]
    shell_sort( scores)
    length = len( scores)
    for i in range( 0 , length):
        print ( scores[ i] , " , " , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

0,1,2,3,4,5,6,7,8,9,

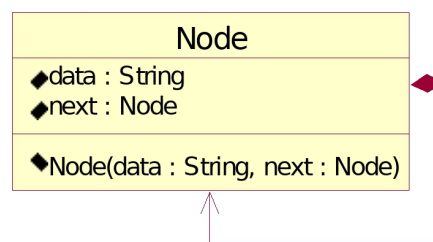
Unidirectional Linked List

Unidirectional Linked List Single Link:

Is a chained storage structure of a linear table, which is connected by a node. Each node consists of data and next pointer to the next node.



UML Diagram



class Node :

data = ""

next = **None**

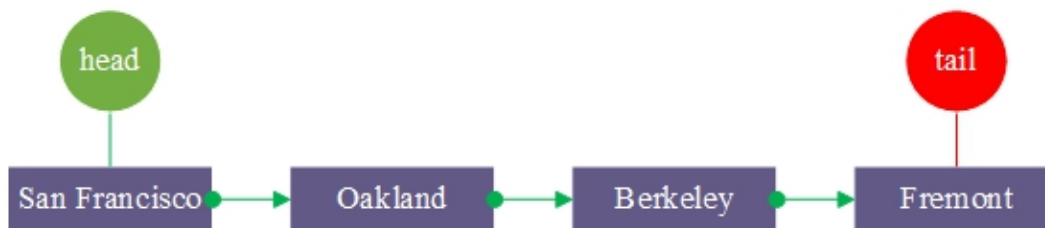
def __init__ (self, data, next):

self.data = data

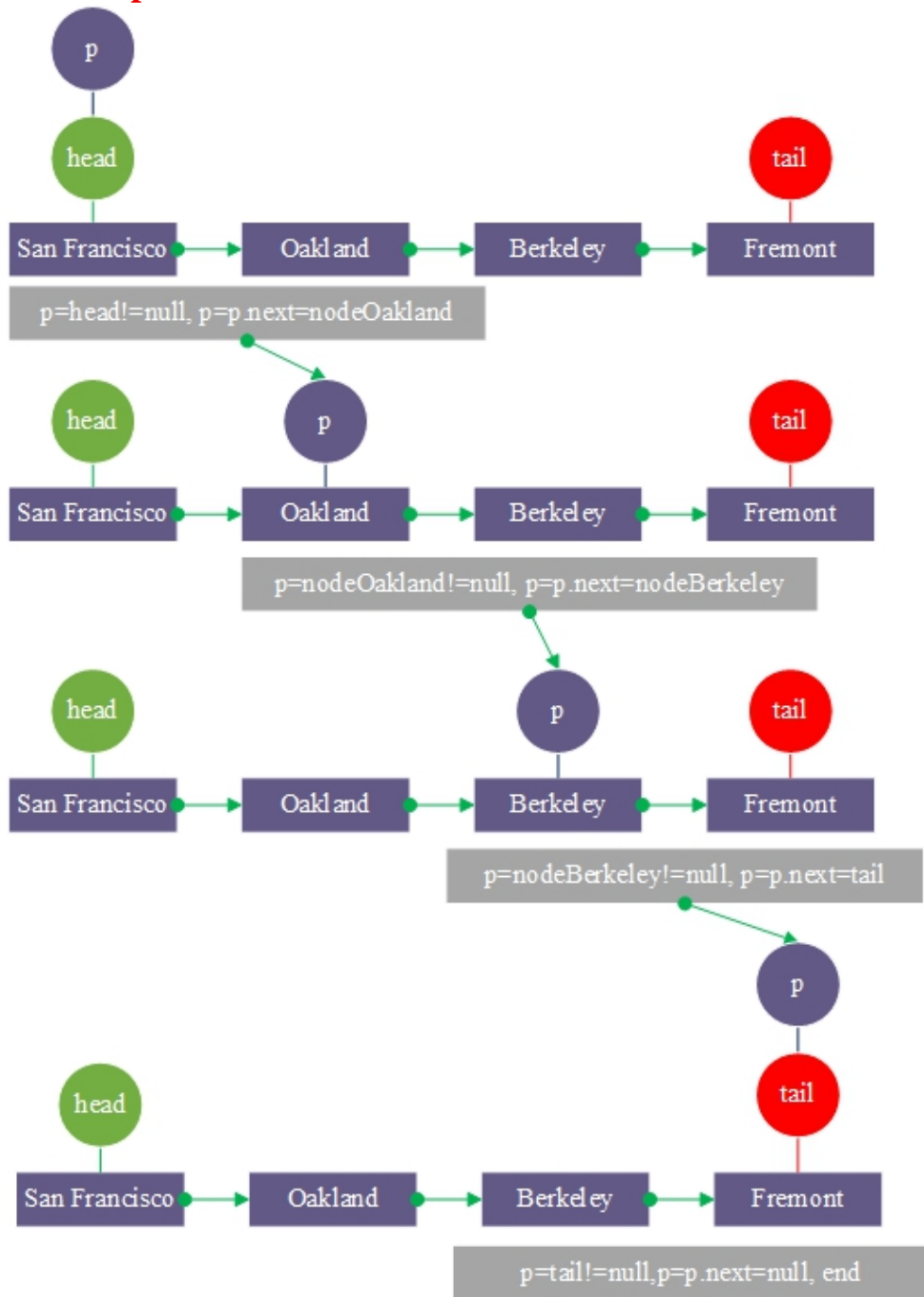
self.next = next

1. Unidirectional Linked List **initialization** .

Example : Construct a San Francisco subway Unidirectional linked list



2. traversal output .



test_unidirectional_linkedlist.py

```
import sys

class Node :
    data = ""
    next = None

    def __init__ ( self, data, next):
        self.data = data
        self.next = next

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        # the first node called head node
        self.__head = Node( "San Francisco" , None )

        node_oakland = Node( "Oakland" , None )
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" , None )
        node_oakland.next = node_berkeley

        # the last node called tail node
        self.__tail = Node( "Fremont" , None )
        node_berkeley.next = self.__tail

        return self.__head

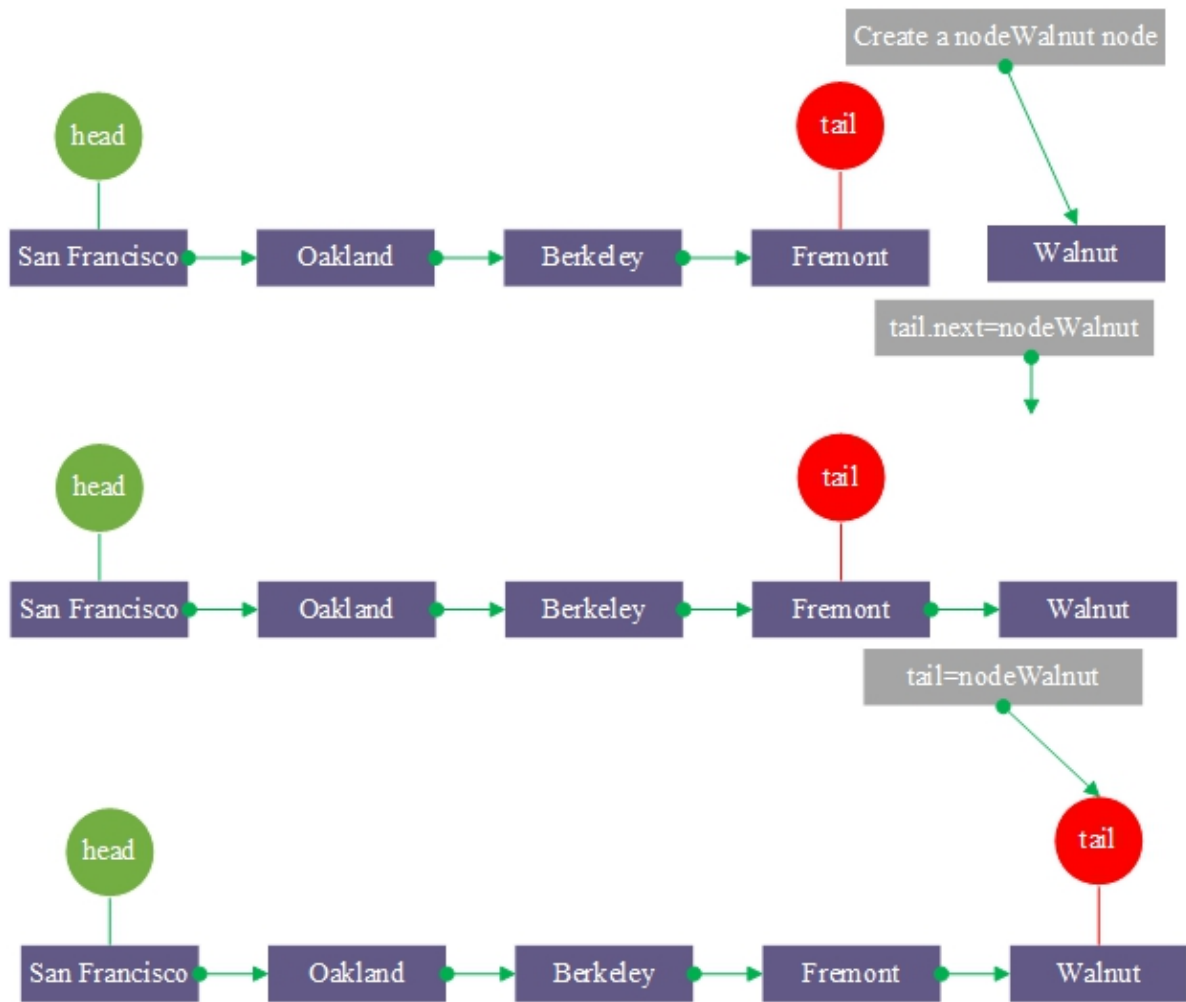
    def output ( self, node):
        p = node
        while ( p != None ): # From the beginning to the end
            data = p.data
            print ( data, " -> " , end= "" )
            p = p.next
        print ( "End\n\n" )
```

```
def main():  
    linkedlist = LinkedList()  
    head = linkedlist.init()  
    linkedlist.output(head)  
  
if __name__ == "__main__":  
    main()
```

Result:

San Francisco -> Oakland -> Berkeley -> Fremont -> End

3. Append a new node name: **Walnut** to the end.



test_unidirectional_linkedlist.py

```
import sys
class Node :
    data = ""
    next = None

    def __init__ ( self, data, next):
        self.data = data
        self.next = next

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        # the first node called head node
        self.__head = Node( "San Francisco" , None )

        node_oakland = Node( "Oakland" , None )
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" , None )
        node_oakland.next = node_berkeley

        # the last node called tail node
        self.__tail = Node( "Fremont" , None )
        node_berkeley.next = self.__tail

        return self.__head

    def add ( self, new_node):
        self.__tail.next = new_node
        self.__tail = new_node

    def output ( self, node):
        p = node
        while ( p != None ): # From the beginning to the end
            data = p.data
```



```
print ( data, " -> ", end= "" )  
p = p. next  
print ( "End\n\n" )
```

```
def main():
    linkedlist = LinkedList()
    head = linkedlist.init()

    print ( "Append a new node name: Walnut  to the end: " )
    linkedlist.add( Node( "Walnut" , None ))

    linkedlist.output( head)

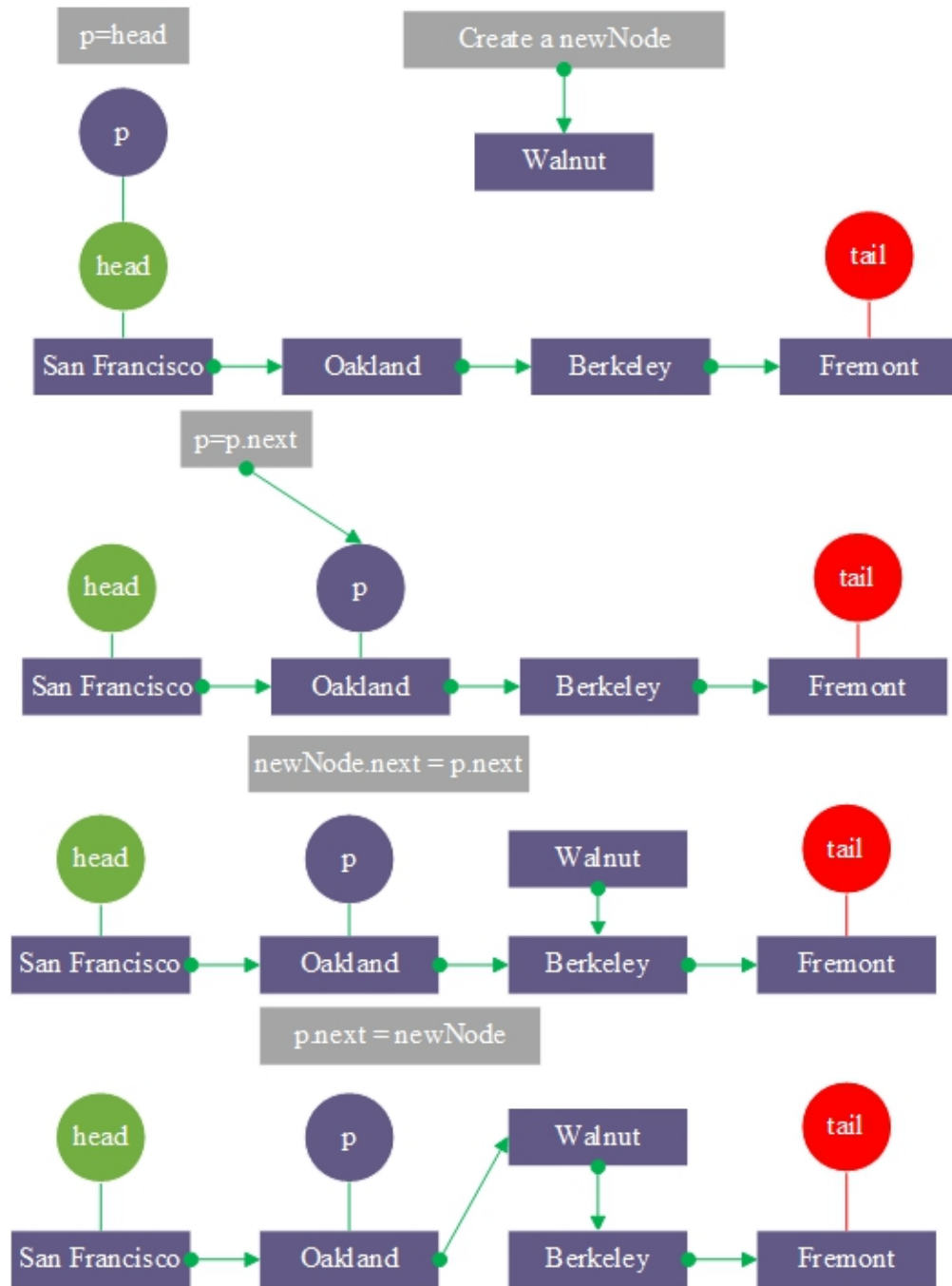
if __name__ == "__main__" :
    main()
```

Result:

Append a new node name: Walnut to the end:

San Francisco -> Oakland -> Berkeley -> Fremont -> Walnut -> End

3. Insert a node **Walnut** in position 2.



test_unidirectional_linkedlist.py

```
import sys
class Node :
    data = ""
    next = None

    def __init__ ( self, data, next):
        self.data = data
        self.next = next

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        # the first node called head node
        self.__head = Node( "San Francisco" , None )

        node_oakland = Node( "Oakland" , None )
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" , None )
        node_oakland.next = node_berkeley

        # the last node called tail node
        self.__tail = Node( "Fremont" , None )
        node_berkeley.next = self.__tail
        return self.__head

    def insert ( self, insert_position, new_node):
        p = self.__head
        i = 0
        # Move the node to the insertion position
        while p.next != None and i < insert_position - 1 :
            p = p.next
            i += 1

        new_node.next = p.next # new_node next point to next node
```

```
p.next = new_node # current next point to new_node
```

```
def output ( self, node):  
    p = node  
    while ( p != None ): # From the beginning to the end  
        data = p.data  
        print ( data, " -> " , end= "" )  
        p = p.next  
    print ( "End\n\n" )
```

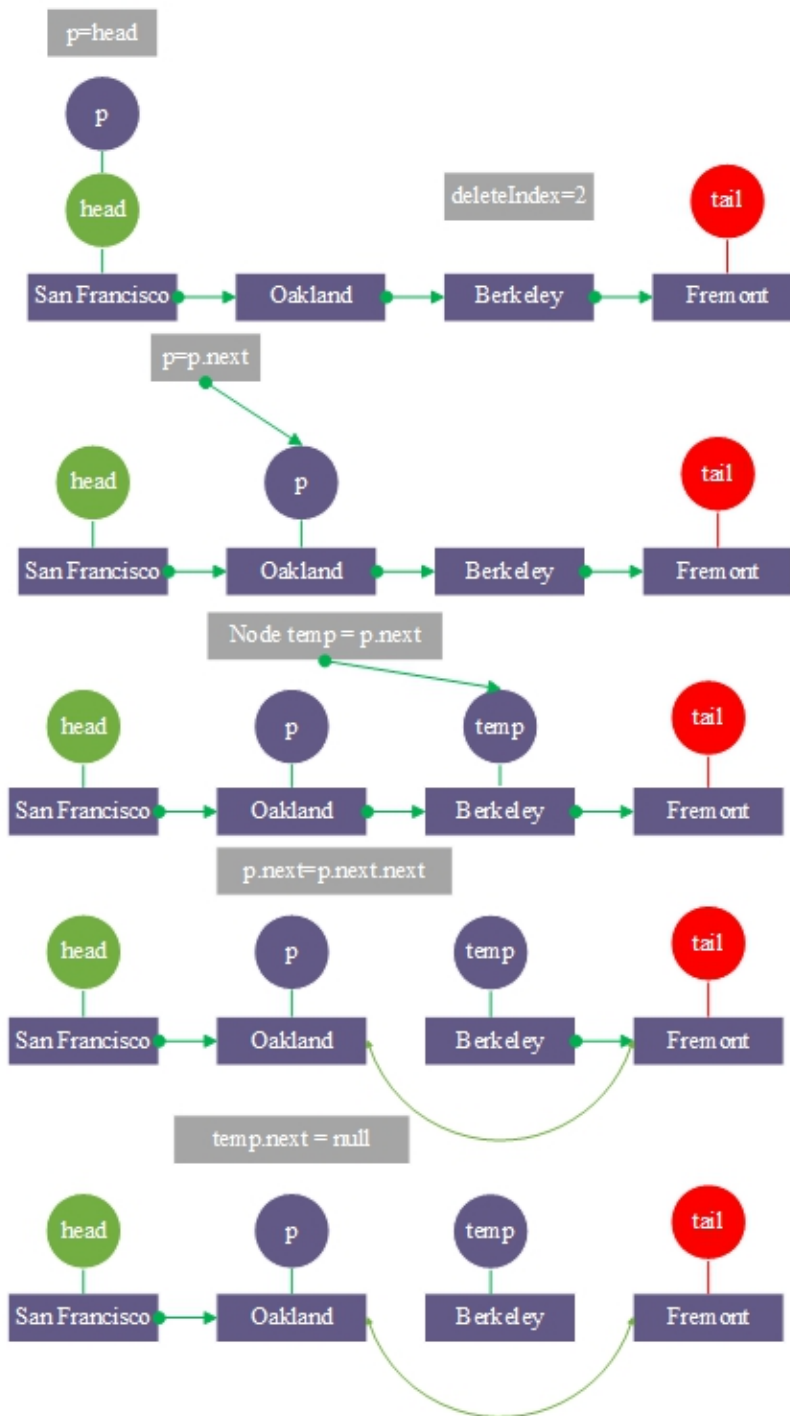
```
def main ():  
    linkedlist = LinkedList()  
    head = linkedlist.init()  
  
    print ( "Insert a new node Walnut at index = 2 : " )  
    linkedlist.insert( 2 , Node( "Walnut" , None ))  
  
    linkedlist.output( head)  
  
if __name__ == "__main__" :  
    main()
```

Result:

Insert a new node Walnut at index = 2 :

San Francisco -> Oakland -> Walnut -> Berkeley -> Fremont -> End

4. Delete the **index=2** node.



test_unidirectional_linkedlist.py

```
import sys
class Node :
    data = ""
    next = None

    def __init__ ( self, data, next):
        self.data = data
        self.next = next

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        # the first node called head node
        self.__head = Node( "San Francisco" , None )

        node_oakland = Node( "Oakland" , None )
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" , None )
        node_oakland.next = node_berkeley

        # the last node called tail node
        self.__tail = Node( "Fremont" , None )
        node_berkeley.next = self.__tail
        return self.__head

    def remove ( self, remove_position):
        p = self.__head
        i = 0
        # Move the node to the previous node position that was deleted
        while p.next != None and i < remove_position - 1 :
            p = p.next
            i += 1
        temp = p.next # Save the node you want to delete
```

```
    p.next = p.next.next # Previous node next points to next of
delete the node
    temp.next = None
```

```
def output ( self, node):
    p = node
    while ( p != None ): # From the beginning to the end
        data = p.data
        print ( data, " -> " , end= "" )
        p = p.next
    print ( "End\n\n" )
```

```
def main ():
    linkedlist = LinkedList()
    head = linkedlist.init()

    print ( "Delete a new node Berkeley at index = 2 : " )
    linkedlist.remove( 2 )

    linkedlist.output( head)

if __name__ == "__main__" :
    main()
```

Result:

Delete a new node Berkeley at index = 2 :
San Francisco -> Oakland -> Fremont -> End

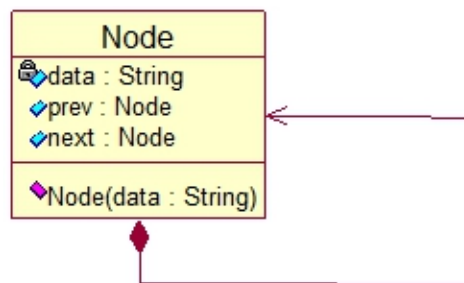
Doubly Linked List

Doubly Linked List:

It is a chained storage structure of a linear table. It is connected by nodes in two directions. Each node consists of data, pointing to the previous node and pointing to the next node.



UML Diagram



```
class Node :
    data = ""
    prev = None
    next = None

    def __init__( self, data):
        self.data = data
```

1. Doubly Linked List **initialization** .

Example : Construct a San Francisco subway Doubly linked list



2. traversal output .

test_doubly_linkedlist.py

```
import sys

class Node :
    data = ""
    prev = None
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "San Francisco" )
        self.__head.prev = None
        self.__head.next = None

        node_oakland = Node( "Oakland" )
        node_oakland.prev = self.__head
        node_oakland.next = None
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" )
        node_berkeley.prev = node_oakland
        node_berkeley.next = None
        node_oakland.next = node_berkeley

        self.__tail = Node( "Fremont" )
        self.__tail.prev = node_berkeley
        self.__tail.next = None
        node_berkeley.next = self.__tail
        return self.__head
```

```

def output ( self, node ):
    p = node
    end = None
    while p != None :
        print ( p. data, " -> " , end= "" )
        end = p
        p = p. next
        print ( "End\n" )

    p = end
    while p != None :
        print ( p. data, " -> " , end= "" )
        p = p. prev
        print ( "Start\n\n" )

def main ():
    linkedlist = LinkedList()
    head = linkedlist. init()
    linkedlist. output( head)

if __name__ == "__main__" :
    main()

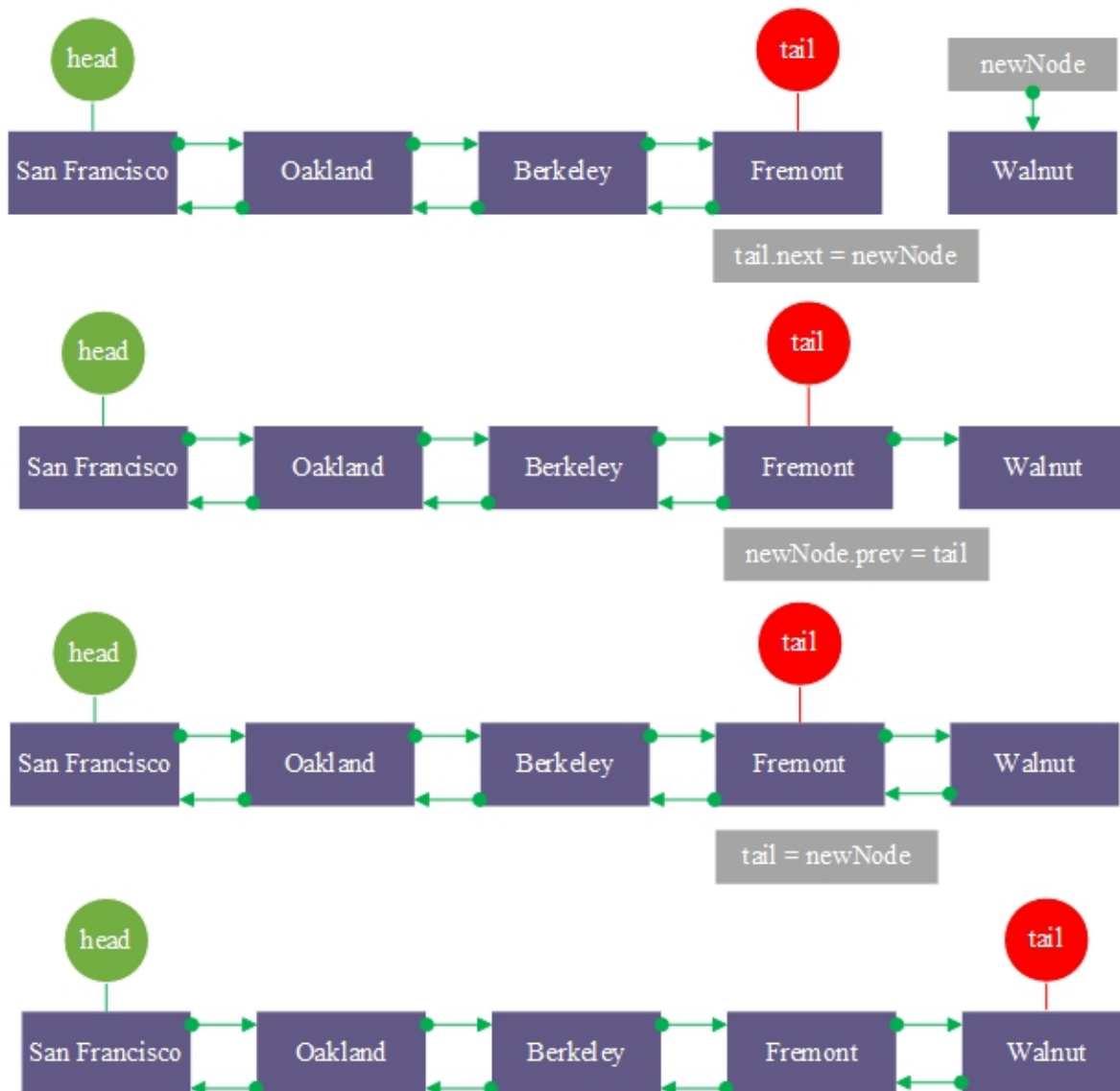
```

Result:

San Francisco -> Oakland -> Berkeley -> Fremont -> End

Fremont -> Berkeley -> Oakland -> San Francisco -> Start

3. add a node **Walnut** at the end of Fremont.



test_doubly_linkedlist.py

```
import sys

class Node :
    data = ""
    prev = None
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :

    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "San Francisco" )
        self.__head.prev = None
        self.__head.next = None

        node_oakland = Node( "Oakland" )
        node_oakland.prev = self.__head
        node_oakland.next = None
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" )
        node_berkeley.prev = node_oakland
        node_berkeley.next = None
        node_oakland.next = node_berkeley

        self.__tail = Node( "Fremont" )
        self.__tail.prev = node_berkeley
        self.__tail.next = None
        node_berkeley.next = self.__tail
        return self.__head

    def add ( self, new_node ):
```

```
self.__tail.next = new_node;
new_node.prev = self.__tail
self.__tail = new_node
```

```
def output ( self, node):
    p = node
    end = None
    while p != None :
        print ( p. data, " -> " , end= "" )
        end = p
        p = p. next
    print ( "End\n" )

    p = end
    while p != None :
        print ( p. data, " -> " , end= "" )
        p = p. prev
    print ( "Start\n\n" )
```

```
def main ():
    linkedlist = LinkedList()
    head = linkedlist. init()

    print ( "Append a new node name: Walnut  to the end: " )
    linkedlist. add( Node( "Walnut" ))

    linkedlist. output( head)

if __name__ == "__main__" :
    main()
```

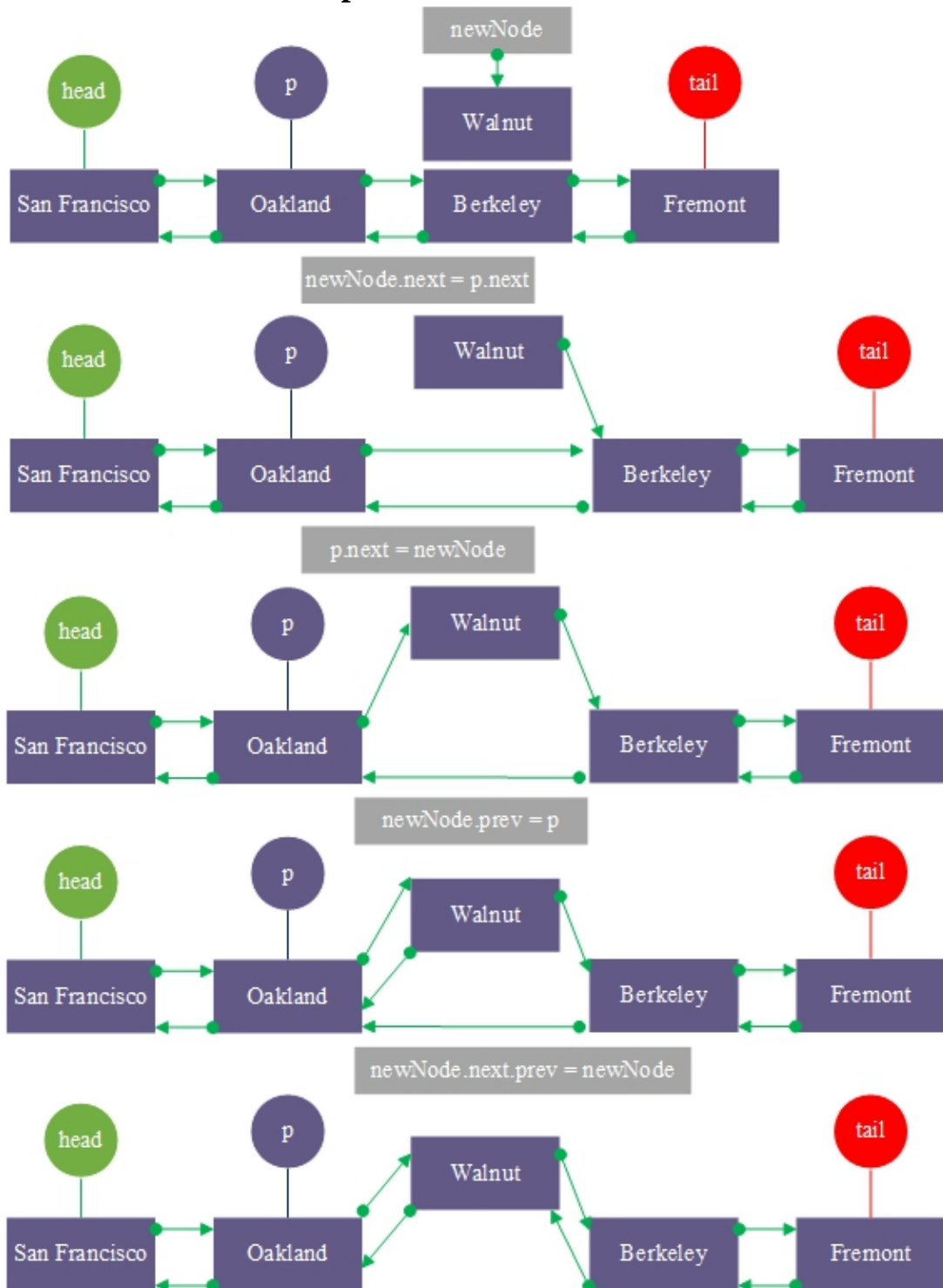
Result:

Append a new node name: Walnut to the end:

San Francisco -> Oakland -> Berkeley -> Fremont -> Walnut -> End

Walnut -> Fremont -> Berkeley -> Oakland -> San Francisco -> Start

3. Insert a node **Walnut** in position 2.



test_doubly_linkedlist.py

```
import sys

class Node :
    data = ""
    prev = None
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :

    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "San Francisco" )
        self.__head.prev = None
        self.__head.next = None

        node_oakland = Node( "Oakland" )
        node_oakland.prev = self.__head
        node_oakland.next = None
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" )
        node_berkeley.prev = node_oakland
        node_berkeley.next = None
        node_oakland.next = node_berkeley

        self.__tail = Node( "Fremont" )
        self.__tail.prev = node_berkeley
        self.__tail.next = None
        node_berkeley.next = self.__tail
        return self.__head
```



```

def insert ( self, insert_position, new_node ):
    p = self. __head
    i = 0
    while p. next != None and i < insert_position- 1 :
        p = p. next
        i+= 1
    new_node. next = p. next
    p. next = new_node
    new_node. prev = p
    new_node. next. prev = new_node

def output ( self, node):
    p = node
    end = None
    while p != None :
        print ( p. data, " -> " , end= "" )
        end = p
        p = p. next
    print ( "End\n" )

    p = end
    while p != None :
        print ( p. data, " -> " , end= "" )
        p = p. prev
    print ( "Start\n\n" )

def main ():
    linkedlist = LinkedList()
    head = linkedlist. init()

    print ( "Insert a new node Walnut at index = 2 : " )
    linkedlist. insert( 2 , Node( "Walnut" ))
    linkedlist. output( head)

if __name__ == "__main__" :
    main()

```

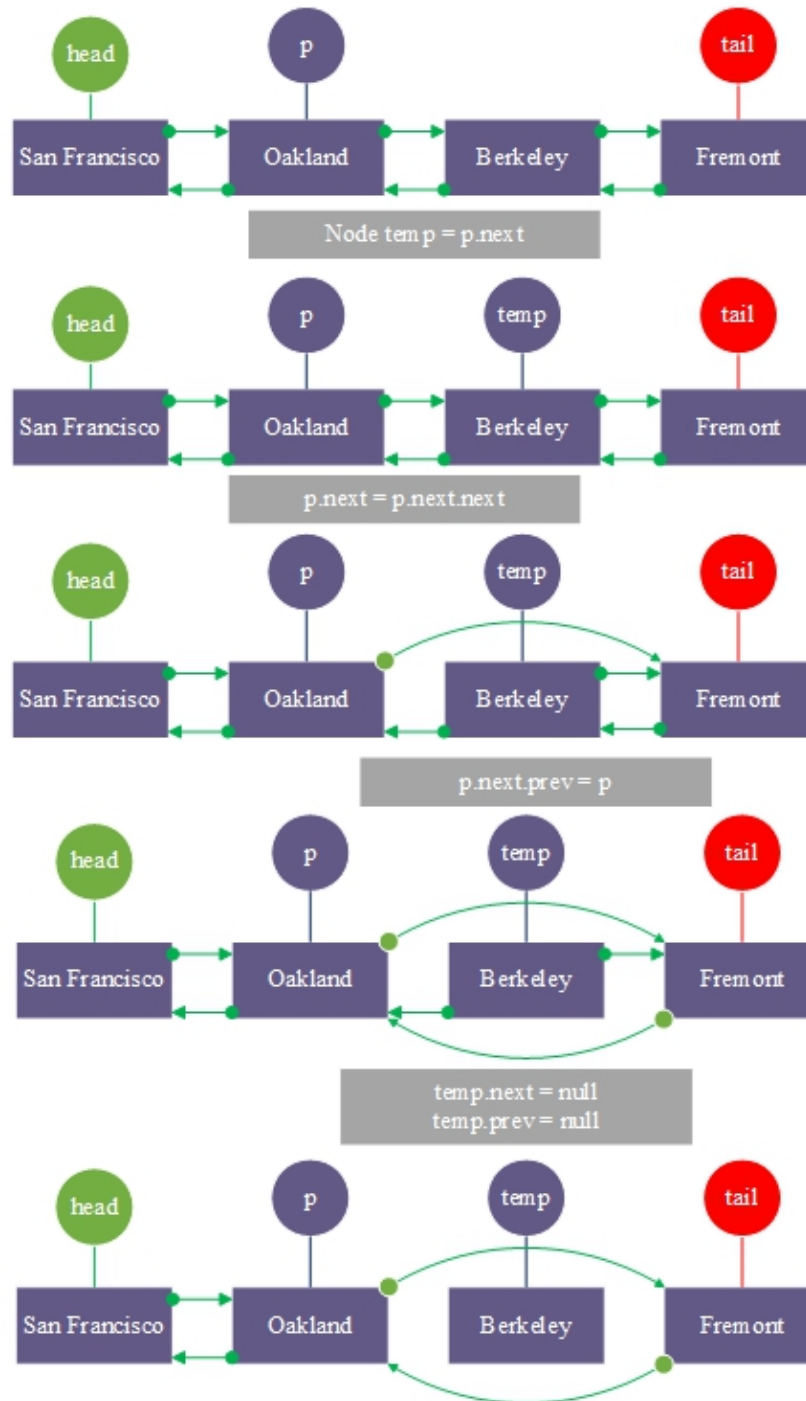
Result:

Insert a new node Walnut at index = 2 :

San Francisco -> Oakland -> Walnut -> Berkeley -> Fremont -> End

Fremont -> Berkeley -> Walnut -> Oakland -> San Francisco -> Start

4. Delete the **index=2** node .



TestDoubleLink.py

```
import sys

class Node :
    data = ""
    prev = None
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :

    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "San Francisco" )
        self.__head.prev = None
        self.__head.next = None

        node_oakland = Node( "Oakland" )
        node_oakland.prev = self.__head
        node_oakland.next = None
        self.__head.next = node_oakland

        node_berkeley = Node( "Berkeley" )
        node_berkeley.prev = node_oakland
        node_berkeley.next = None
        node_oakland.next = node_berkeley

        self.__tail = Node( "Fremont" )
        self.__tail.prev = node_berkeley
        self.__tail.next = None
        node_berkeley.next = self.__tail
        return self.__head

    def remove ( self, remove_position ):
```

```

p = self.__head
i = 0
# Move the node to the previous node that was deleted
while p.next != None and i < remove_position - 1 :
    p = p.next
    i += 1

temp = p.next # Save the node you want to delete
p.next = p.next.next
p.next.prev = p
temp.next = None # Set the delete node next to null
temp.prev = None # Set the delete node prev to null

```

```

def output ( self, node):
    p = node
    end = None
    while p != None :
        print ( p.data, " -> ", end= "" )
        end = p
        p = p.next
    print ( "End\n" )

    p = end
    while p != None :
        print ( p.data, " -> ", end= "" )
        p = p.prev
    print ( "Start\n\n" )

```

```

def main ():
    linkedlist = LinkedList()
    head = linkedlist.init()

    print ( "Delete node Berkeley at index = 2 : " )
    linkedlist.remove( 2 )

    linkedlist.output( head)

if __name__ == "__main__" :
    main ()

```

Result:

Delete node Berkeley at index = 2 :

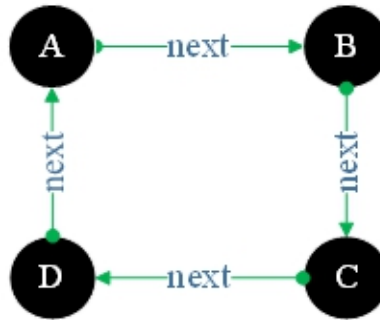
San Francisco -> Oakland -> Fremont -> End

Fremont -> Oakland -> San Francisco -> Start

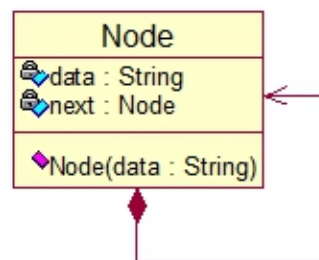
One-way Circular LinkedList

One-way Circular List:

It is a chain storage structure of a linear table, which is connected to form a ring, and each node is composed of data and a pointer to next.



UML Diagram



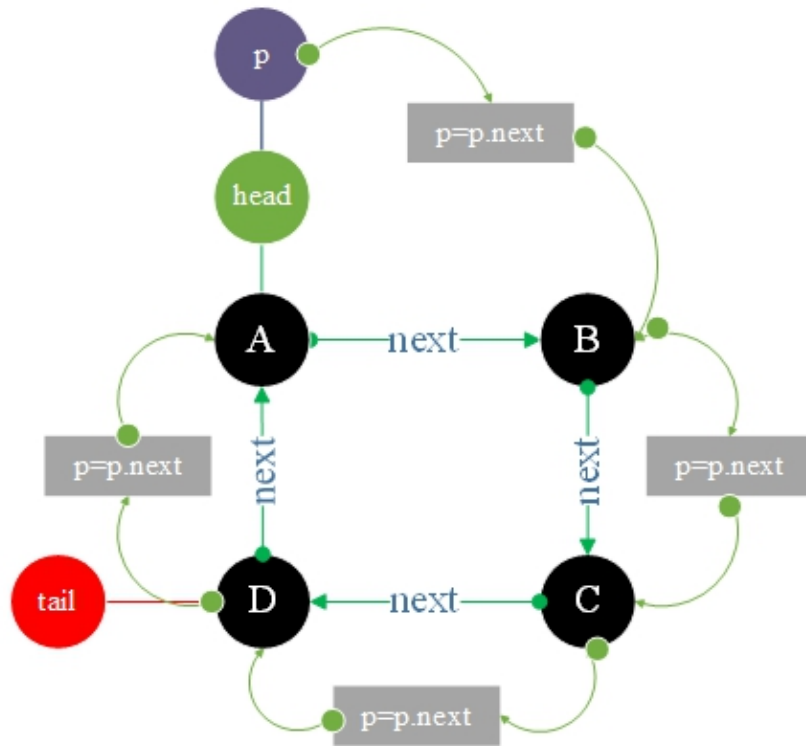
```
class Node :
```

```
    data = "
```

```
    next = None
```

```
    def __init__( self, data):  
        self.data = data
```

1. One-way Circular Linked List **initialization and traversal output** .



test_oneway_circular_linkedlist.py

```
import sys
class Node :

    data = ""
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        #the first node called head node
        self.__head = Node( "A" )
        self.__head.next = None

        nodeB = Node( "B" )
        nodeB.next = None
        self.__head.next = nodeB

        nodeC = Node( "C" )
        nodeC.next = None
        nodeB.next = nodeC

        # the last node called tail node
        self.__tail = Node( "D" )
        self.__tail.next = self.__head
        nodeC.next = self.__tail

    def output ( self):
        p = self.__head
        print ( p.data, " -> ", end= "" )
        p = p.next
        while p != self.__head:
            print ( p.data, " -> ", end= "" )
```

```
        p = p.next

        print ( p.data, "\n\n" )

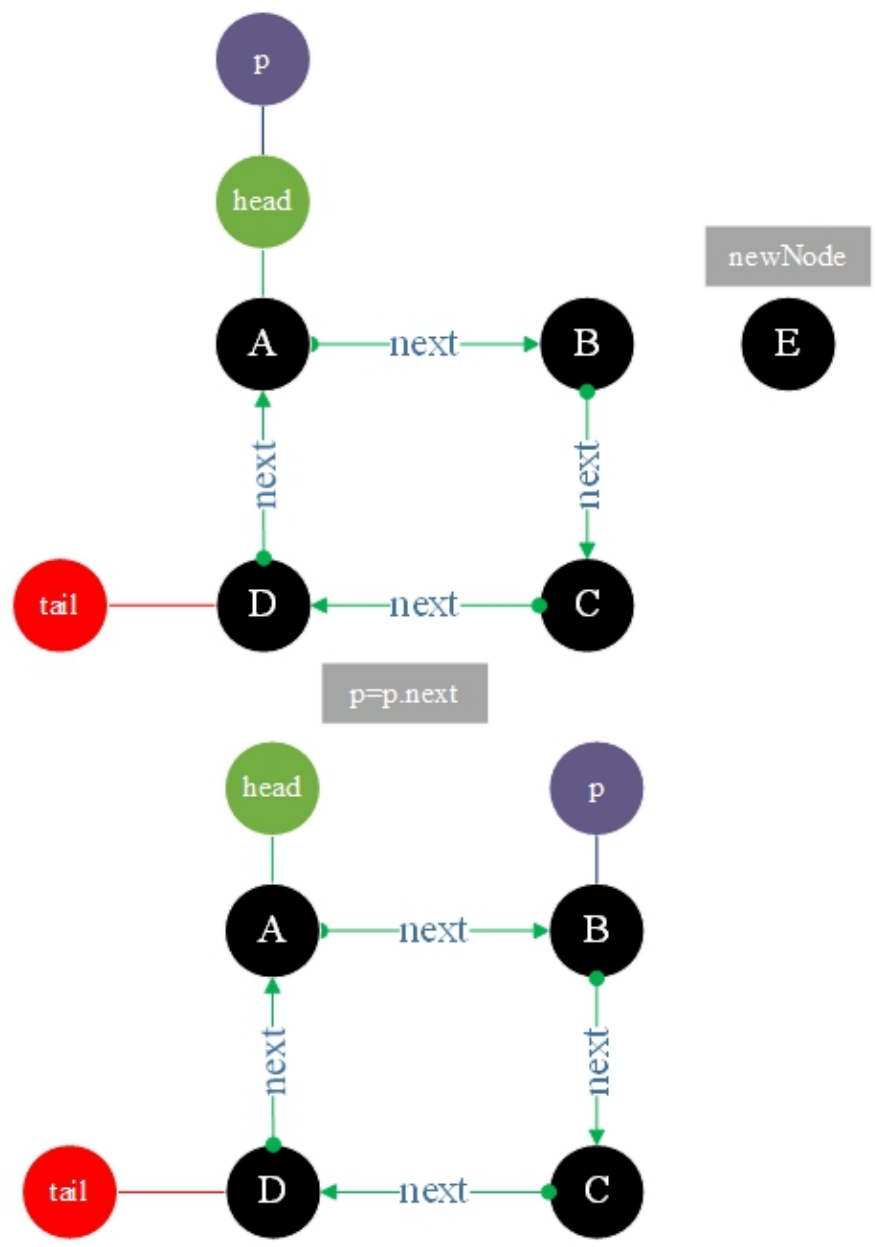
def main ():
    linkedlist = LinkedList()
    linkedlist.init()
    linkedlist.output()

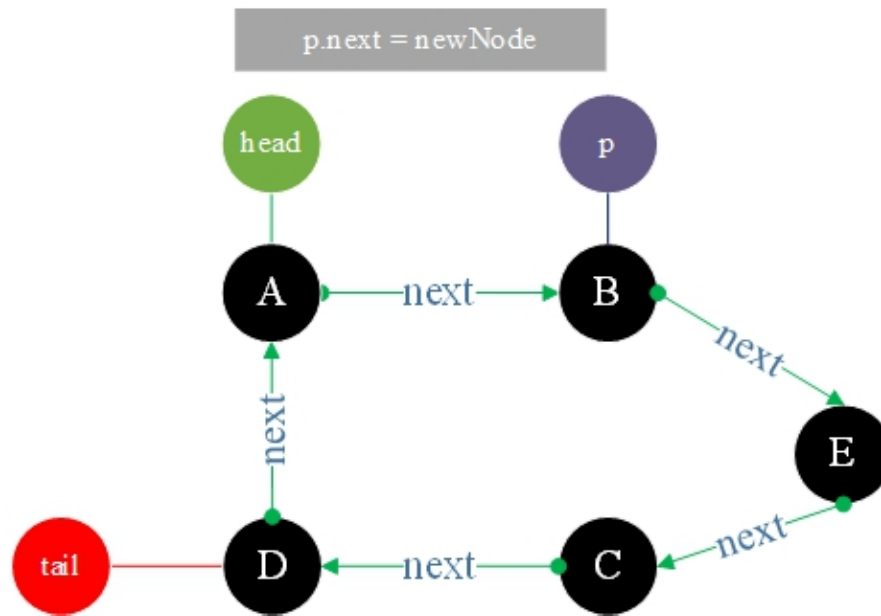
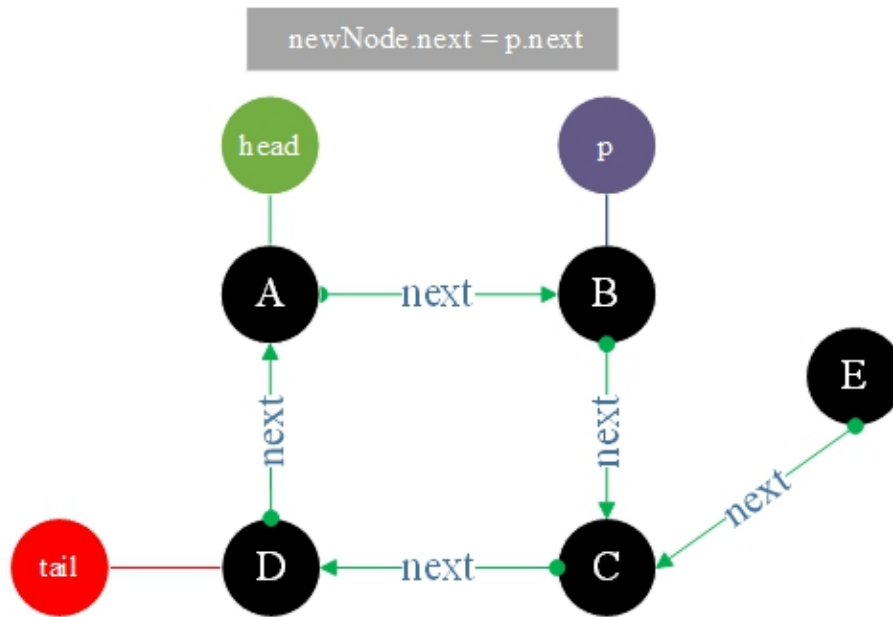
if __name__ == "__main__" :
    main()
```

Result:

A -> B -> C -> D -> A

3. Insert a node **E** in position 2 .





test_oneway_circular_linkedlist.py

```
import sys
class Node :

    data = ""
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "A" )
        self.__head.next = None

        nodeB = Node( "B" )
        nodeB.next = None
        self.__head.next = nodeB

        nodeC = Node( "C" )
        nodeC.next = None
        nodeB.next = nodeC

        self.__tail = Node( "D" )
        self.__tail.next = self.__head
        nodeC.next = self.__tail

    def insert ( self, insert_position, new_node):
        p = self.__head
        i = 0
        # Move the node to the insertion position
        while p.next != None and i < insert_position - 1 :
            p = p.next
            i += 1
        new_node.next = p.next
```

```
p.next = new_node
```

```
def output ( self):  
    p = self.__head  
    print ( p.data, " -> " , end= "" )  
    p = p.next  
    while p != self.__head:  
        print ( p.data, " -> " , end= "" )  
        p = p.next  
    print ( p.data, "\n\n" )
```

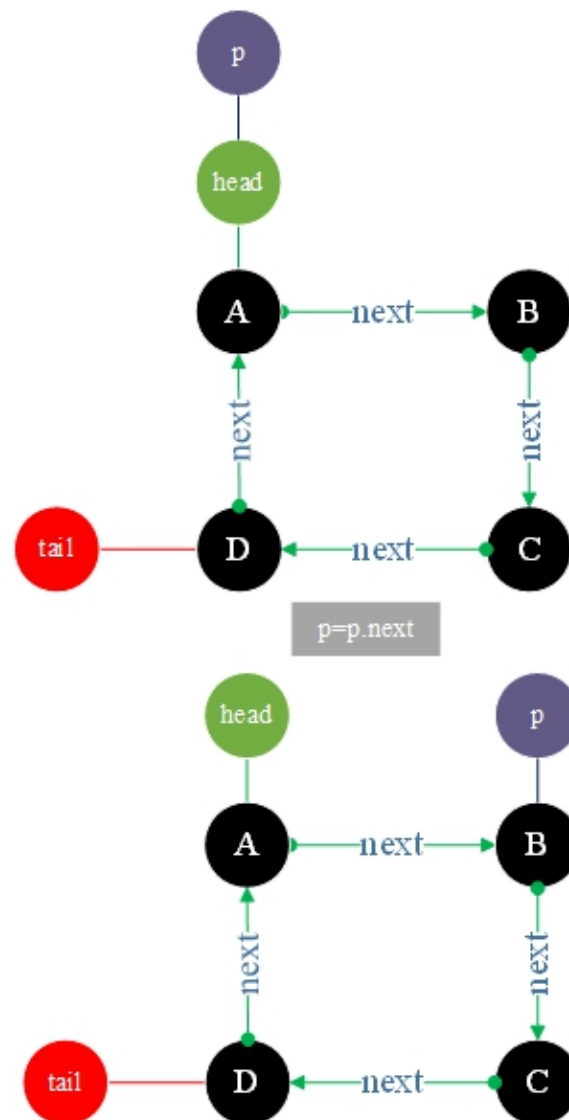
```
def main ():  
    linkedlist = LinkedList()  
    linkedlist.init()  
  
    print ( "Insert a new node E at index = 2 : " )  
    linkedlist.insert( 2 , Node( "E" ))  
  
    linkedlist.output()  
  
if __name__ == "__main__" :  
    main()
```

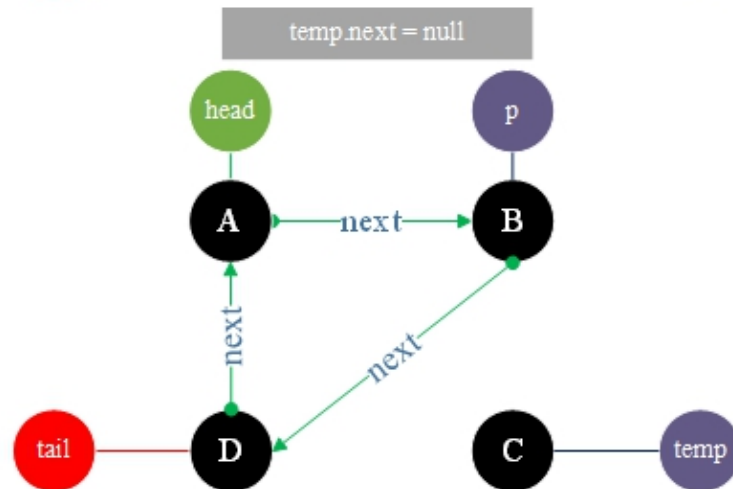
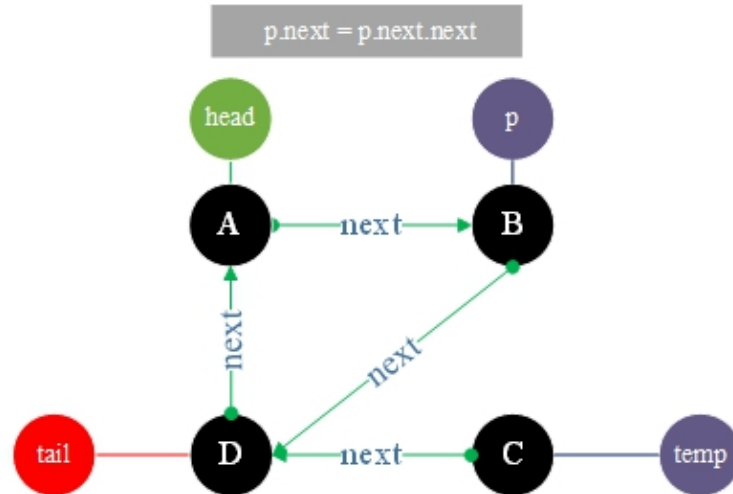
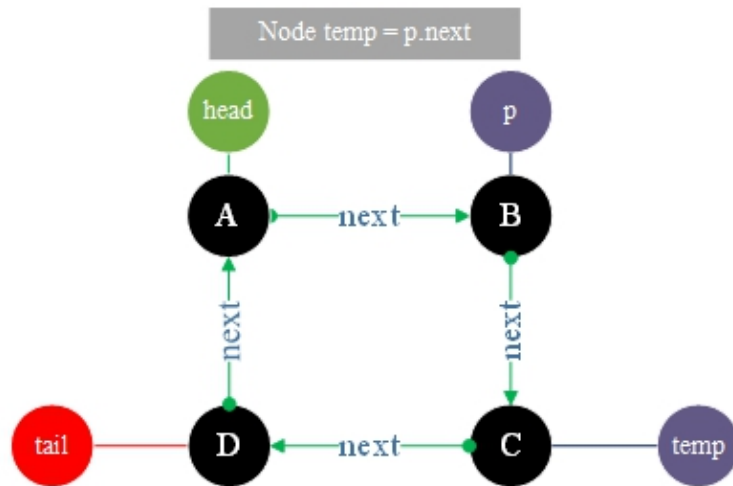
Result:

Insert a new node E at index = 2 :

A -> B -> E -> C -> D -> A

4. Delete the **index=2** node .





test_oneway_circular_linkedlist.py

```
import sys
class Node :

    data = ""
    next = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :

    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "A" )
        self.__head.next = None

        nodeB = Node( "B" )
        nodeB.next = None
        self.__head.next = nodeB

        nodeC = Node( "C" )
        nodeC.next = None
        nodeB.next = nodeC

        self.__tail = Node( "D" )
        self.__tail.next = self.__head
        nodeC.next = self.__tail

    def remove ( self, remove_position):
        p = self.__head
        i = 0
        while p.next != None and i < remove_position - 1 :
            p = p.next
            i += 1
        temp = p.next
        p.next = p.next.next
```

```
temp.next = None
```

```
def output ( self):  
    p = self.__head  
    print ( p.data, " -> " , end= "" )  
    p = p.next  
    while p != self.__head:  
        print ( p.data, " -> " , end= "" )  
        p = p.next  
    print ( p.data, "\n\n" )
```

```
def main ():  
    linkedlist = LinkedList()  
    linkedlist.init()  
  
    print ( "Delete a node at index = 2 : " )  
    linkedlist.remove( 2 )  
  
    linkedlist.output()  
  
if __name__ == "__main__" :  
    main()
```

Result:

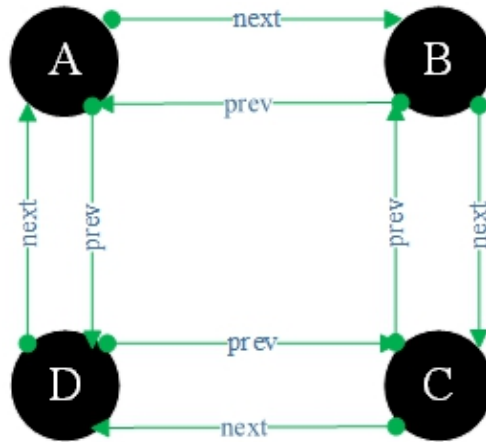
Delete a node at index = 2 :

A -> B -> D -> A

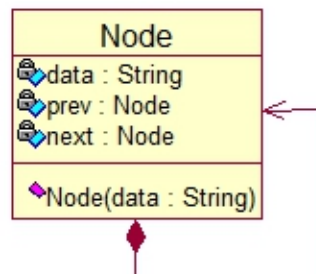
Two-way Circular LinkedList

Two-way Circular List:

It is a chain storage structure of a linear table. The nodes are connected in series by two directions, and is connected to form a ring. Each node is composed of **data** , pointing to the previous node **prev** and pointing to the next node **next** .



UML Diagram



class Node :

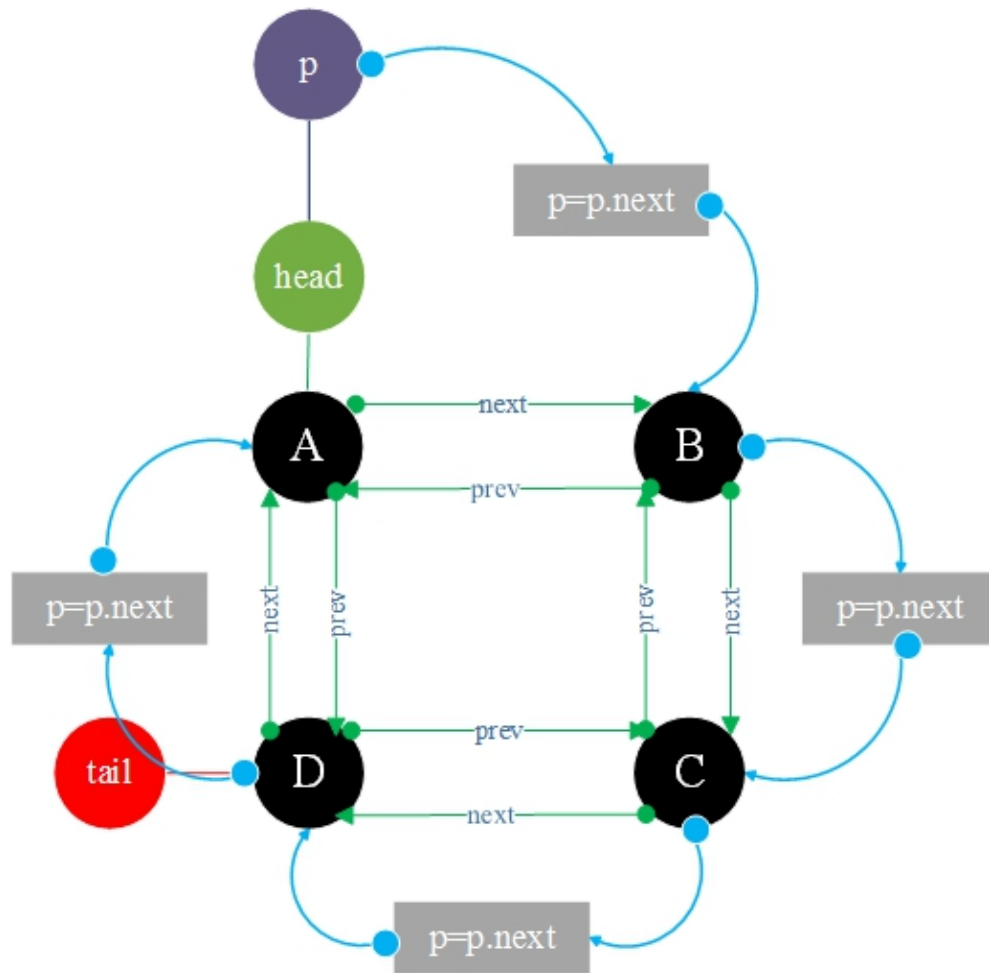
```
data = ""
```

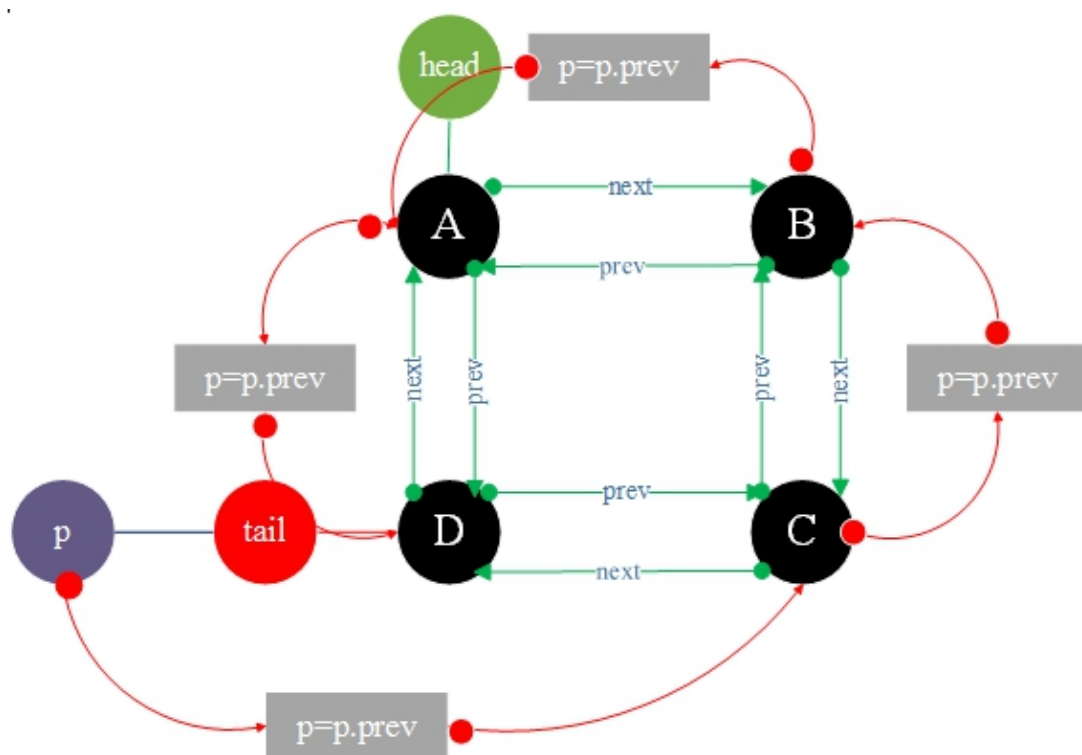
```
next = None
```

```
prev = None
```

```
def __init__( self, data):  
    self.data = data
```

1. Two-way Circular Linked List **initialization and traversal output** .





test_twoway_circular_linkedlist.py

```
import sys

class Node :
    data = ""
    next = None
    prev = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "A" )
        self.__head.next = None
        self.__head.prev = None

        nodeB = Node( "B" )
        nodeB.next = None
        nodeB.prev = self.__head
        self.__head.next = nodeB

        nodeC = Node( "C" )
        nodeC.next = None
        nodeC.prev = nodeB
        nodeB.next = nodeC

        self.__tail = Node( "D" )
        self.__tail.next = self.__head
        self.__tail.prev = nodeC
        nodeC.next = self.__tail
        self.__head.prev = self.__tail
```

```

def output ( self ):
    p = self. __head
    print ( p. data, " -> " , end= "" )
    p = p. next;
    while p != self. __head:
        print ( p. data, " -> " , end= "" )
        p = p. next
        print ( p. data, "\n\n" , end= "" )

    p = self. __tail
    print ( p. data, " -> " , end= "" )
    p = p. prev
    while p != self. __tail:
        print ( p. data, " -> " , end= "" )
        p = p. prev

        print ( p. data, "\n\n" , end= "" )

def main ():
    linkedlist = LinkedList()
    linkedlist. init()
    linkedlist. output()

if __name__ == "__main__" :
    main()

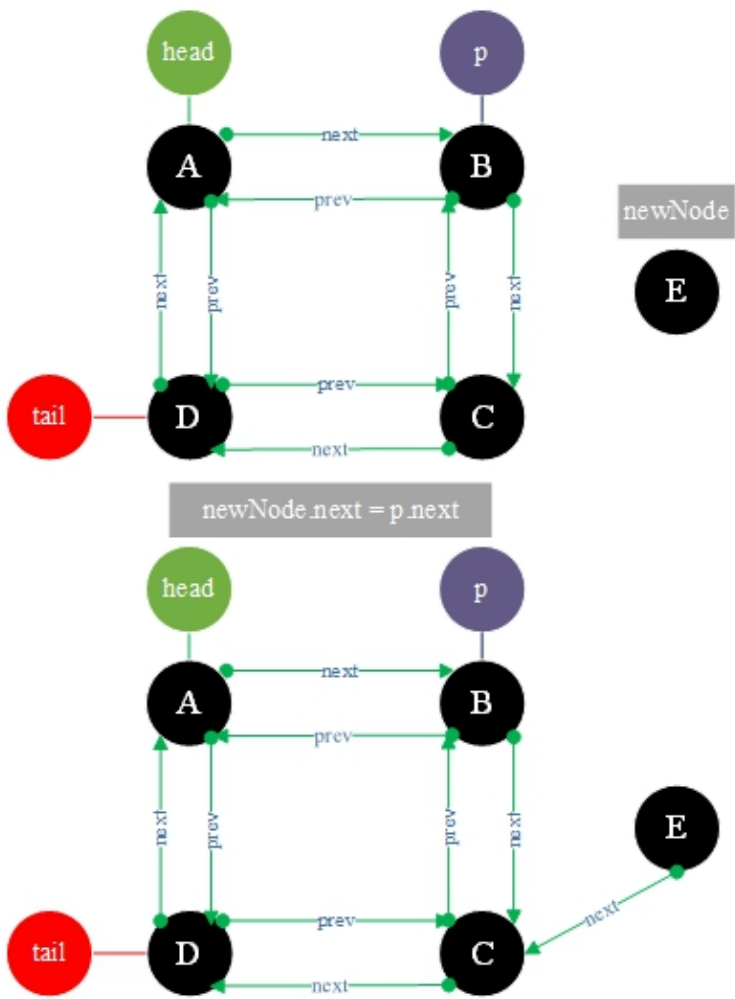
```

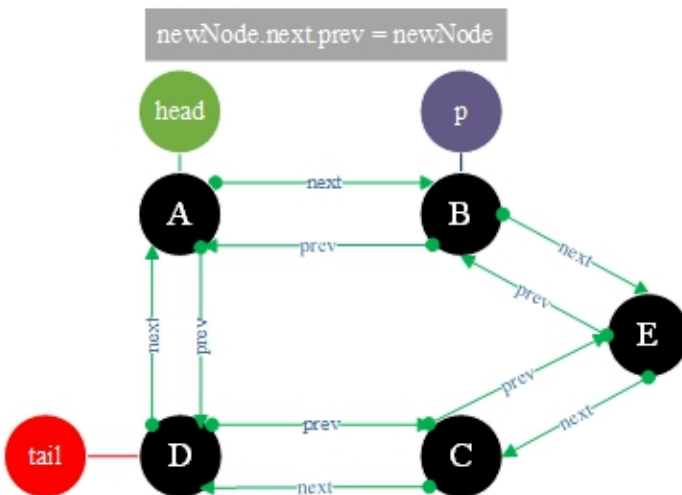
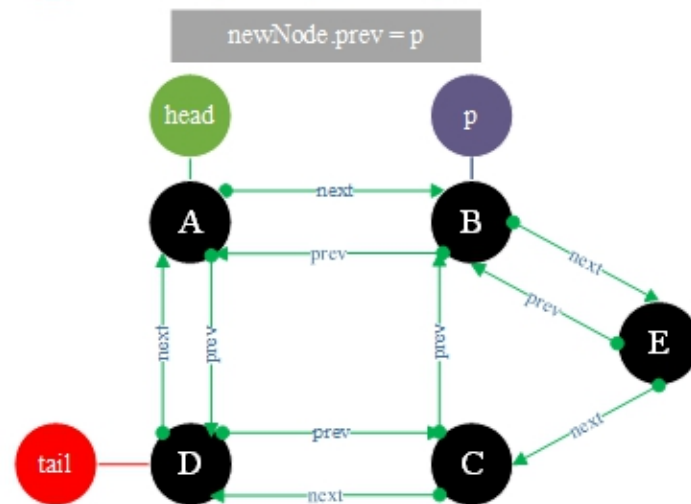
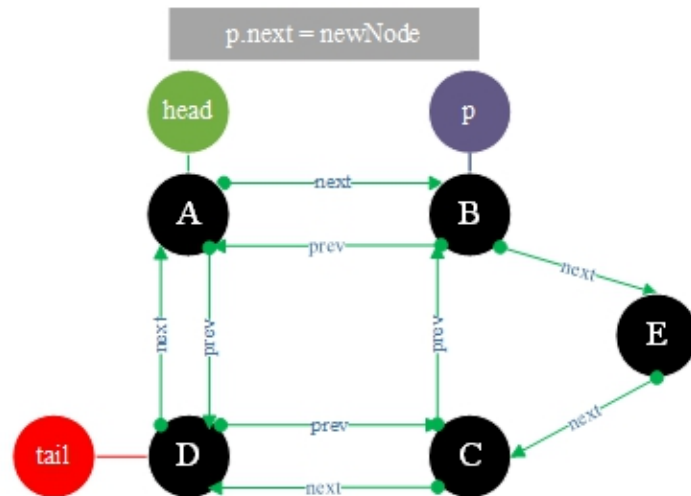
Result:

A -> B -> C -> D -> A

D -> C -> B -> A -> D

3. Insert a node **E** in position 2.





test_twoway_circular_linkedlist.py

```
import sys

class Node :
    data = ""
    next = None
    prev = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "A" )
        self.__head.next = None
        self.__head.prev = None

        nodeB = Node( "B" )
        nodeB.next = None
        nodeB.prev = self.__head
        self.__head.next = nodeB

        nodeC = Node( "C" )
        nodeC.next = None
        nodeC.prev = nodeB
        nodeB.next = nodeC

        self.__tail = Node( "D" )
        self.__tail.next = self.__head
        self.__tail.prev = nodeC
        nodeC.next = self.__tail
        self.__head.prev = self.__tail
```

```

def insert ( self, insert_position, new_node ):
    p = self. __head
    i = 0
    #Move the node to the insertion position
    while p. next != None and i < insert_position - 1 :
        p = p. next
        i += 1
    new_node. next = p. next
    p. next = new_node
    new_node. prev = p
    new_node. next. prev = new_node

```

```

def output ( self):
    p = self. __head
    print ( p. data, " -> " , end= "" )
    p = p. next;
    while p != self. __head:
        print ( p. data, " -> " , end= "" )
        p = p. next
    print ( p. data, "\n\n" , end= "" )

    p = self. __tail
    print ( p. data, " -> " , end= "" )
    p = p. prev
    while p != self. __tail:
        print ( p. data, " -> " , end= "" )
        p = p. prev
    print ( p. data, "\n\n" , end= "" )

```

```

def main ():
    linkedlist = LinkedList()
    linkedlist. init()
    print ( "Insert a new node E at index = 2 : " )
    linkedlist. insert( 2 , Node( "E" ))
    linkedlist. output()

if __name__ == "__main__" :
    main()

```

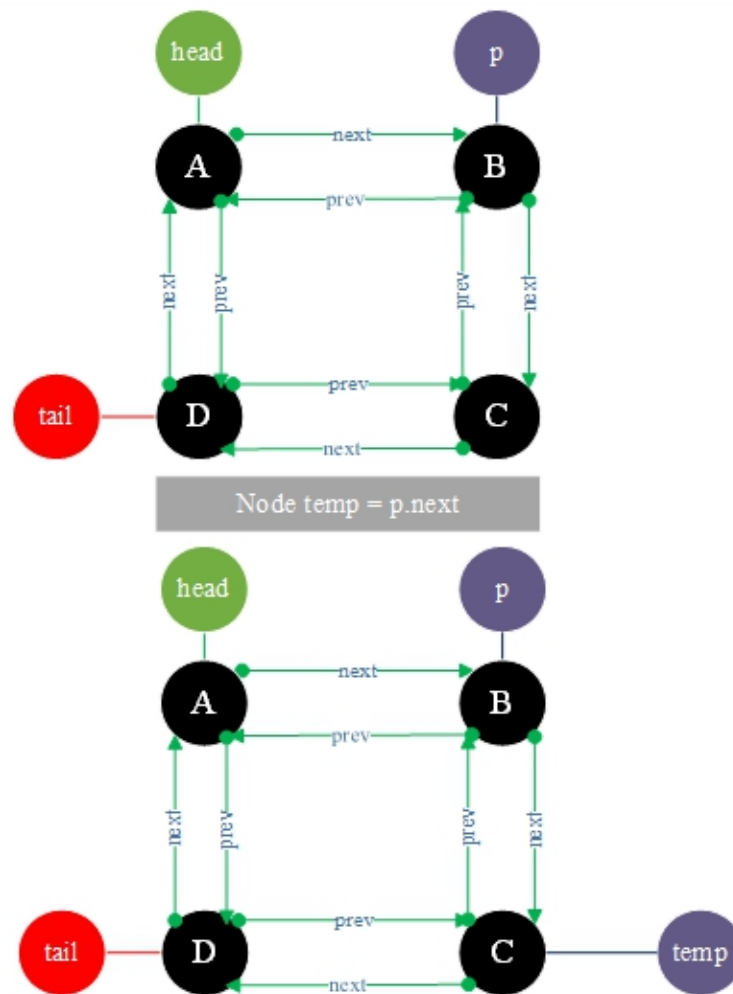
Result:

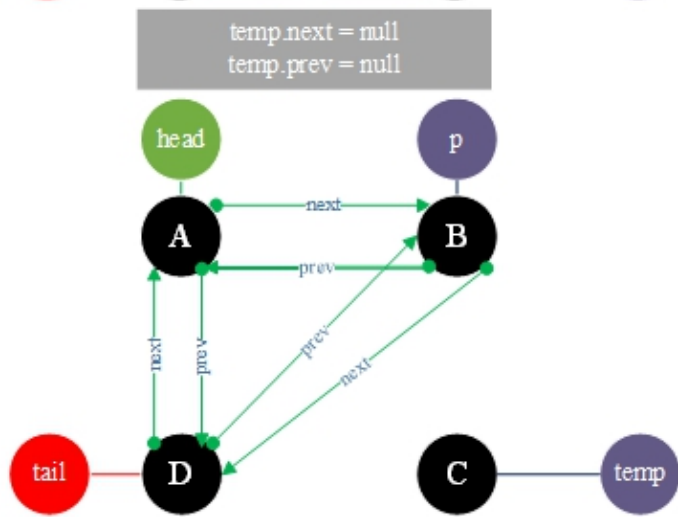
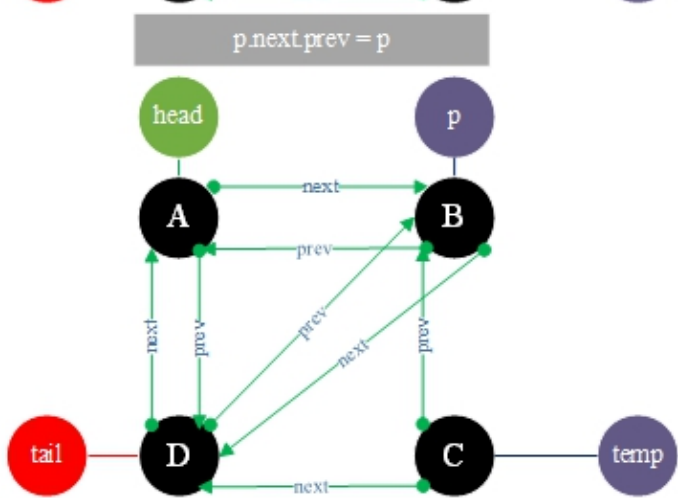
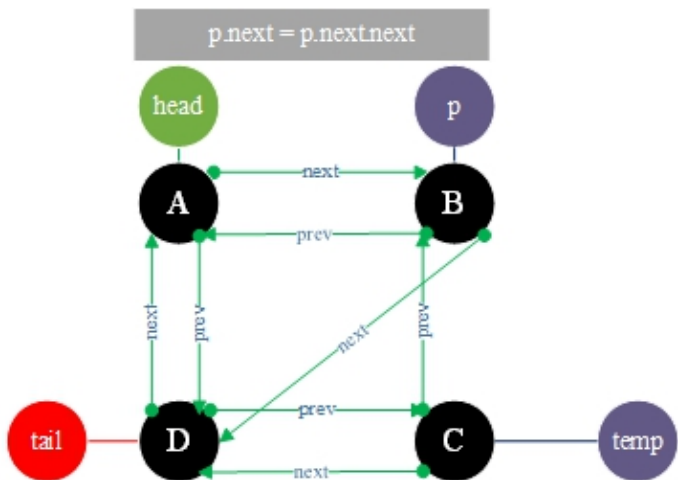
Insert a new node E at index = 2 :

A -> B -> E -> C -> D -> A

D -> C -> E -> B -> A -> D

4. Delete the **index=2** node.





test_twoway_circular_linkedlist.py

```
import sys

class Node :
    data = ""
    next = None
    prev = None

    def __init__ ( self, data):
        self.data = data

class LinkedList :
    __head = None
    __tail = None

    def init ( self):
        self.__head = Node( "A" )
        self.__head.next = None
        self.__head.prev = None

        nodeB = Node( "B" )
        nodeB.next = None
        nodeB.prev = self.__head
        self.__head.next = nodeB

        nodeC = Node( "C" )
        nodeC.next = None
        nodeC.prev = nodeB
        nodeB.next = nodeC

        self.__tail = Node( "D" )
        self.__tail.next = self.__head
        self.__tail.prev = nodeC
        nodeC.next = self.__tail
        self.__head.prev = self.__tail
```

```

def remove ( self, remove_position ):
    p = self. __head
    i = 0
    while p. next != None and i < remove_position - 1 :
        p = p. next
        i += 1
    temp = p. next
    p. next = p. next. next
    p. next. prev = p
    temp. next = None # he delete node next to null
    temp. prev = None # Set the delete node prev to null

```

```

def output ( self):
    p = self. __head
    print ( p. data, " -> " , end= "" )
    p = p. next;
    while p != self. __head:
        print ( p. data, " -> " , end= "" )
        p = p. next
    print ( p. data, "\n\n" , end= "" )

    p = self. __tail
    print ( p. data, " -> " , end= "" )
    p = p. prev
    while p != self. __tail:
        print ( p. data, " -> " , end= "" )
        p = p. prev
    print ( p. data, "\n\n" , end= "" )

```

```

def main ():
    linkedlist = LinkedList()
    linkedlist. init()
    print ( "Delet a node C at index = 2 : " )
    linkedlist. remove( 2 )
    linkedlist. output()

if __name__ == "__main__" :
    main()

```


Result:

Delet a node C at index = 2 :

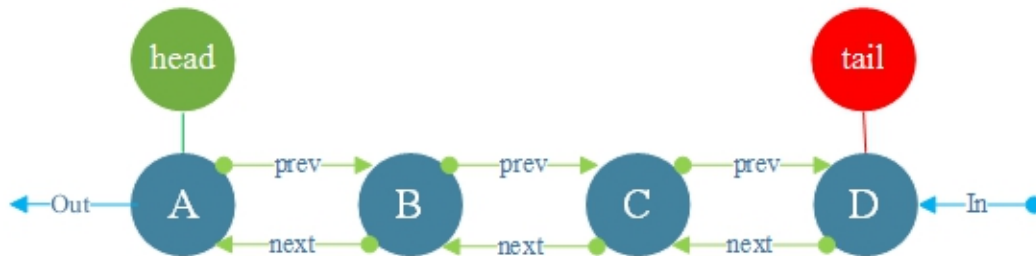
A -> B -> D -> A

D -> B -> A -> D

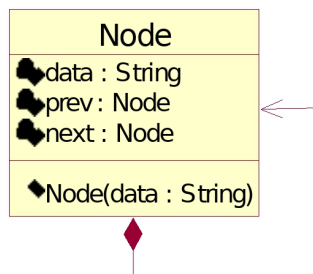
Queue

Queue:

FIFO (First In First Out) sequence.



UML Diagram



class Node :

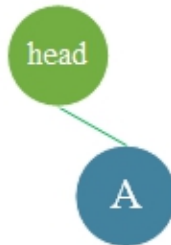
```
data = ""
next = None
prev = None
```

```
def __init__( self, data):
    self. data = data
```

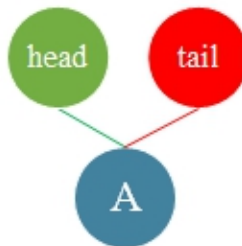
1. Queue **initialization and traversal output** .

Initialization Insert **A**

```
head = new Node( "A" );
```

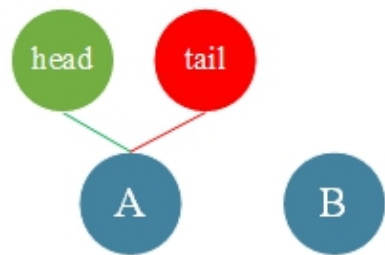


```
tail = head;
```

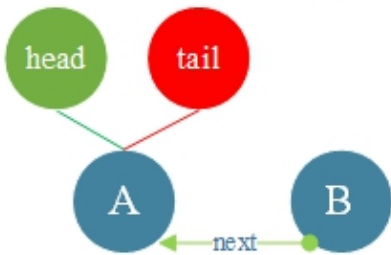


Initialization Insert B

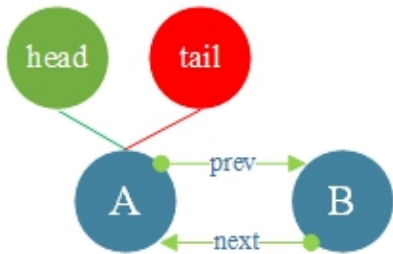
```
newNode = new Node( "B" );
```



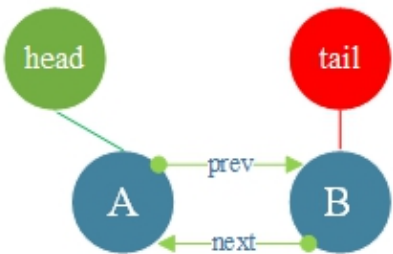
```
newNode.next = tail;
```



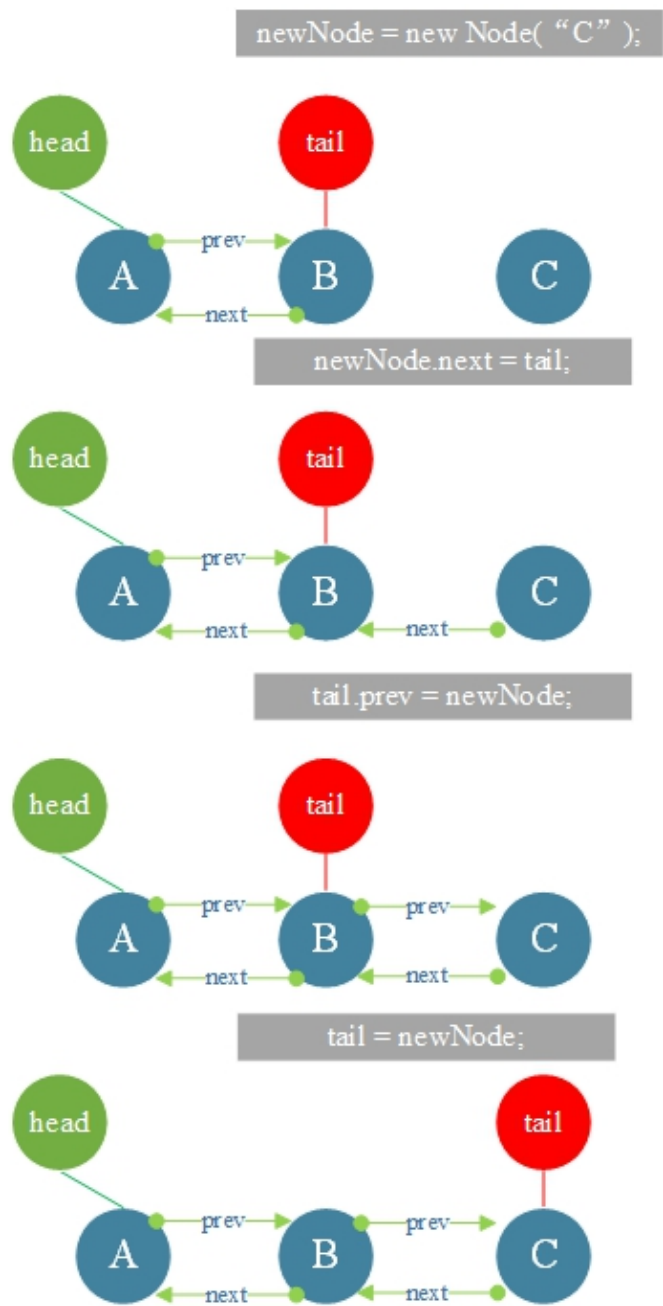
```
tail.prev = newNode;
```



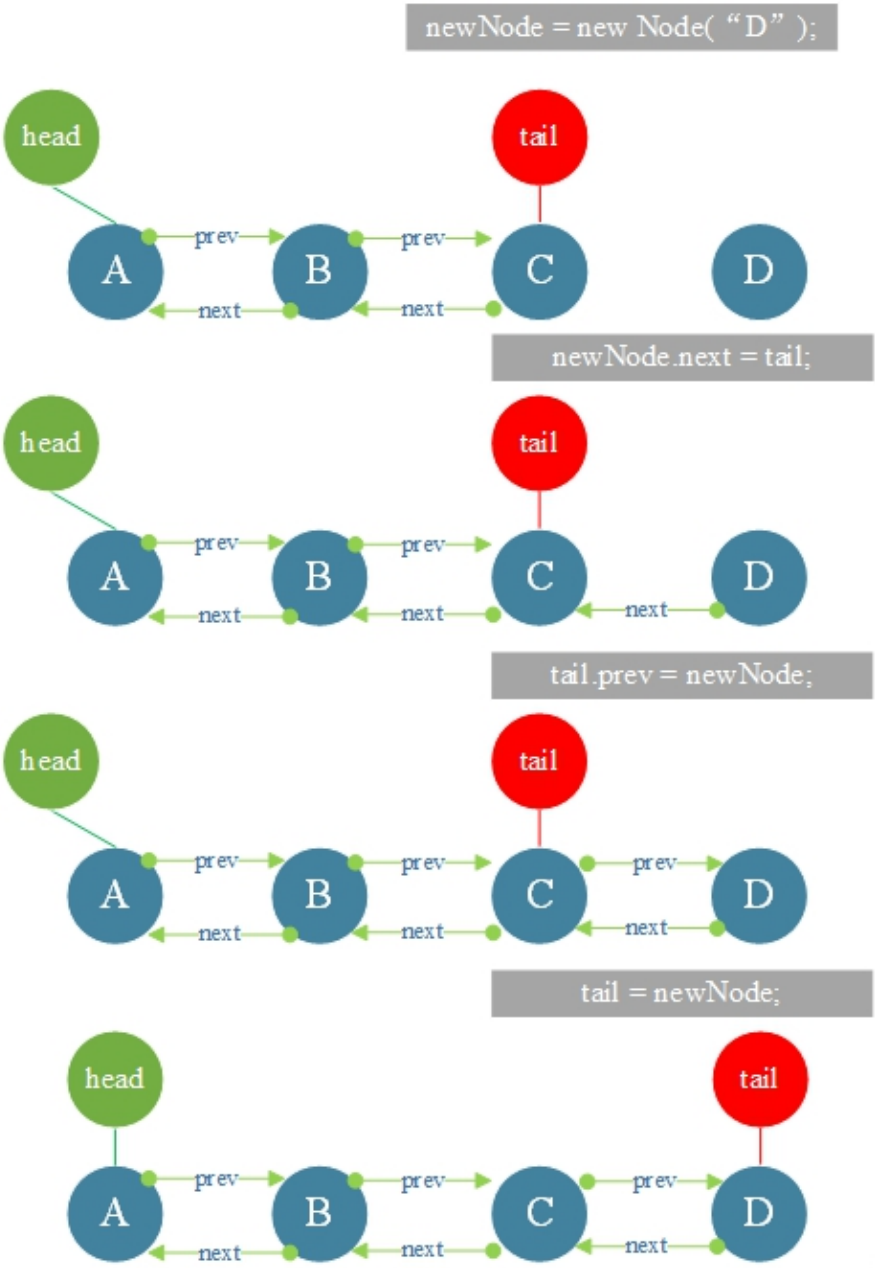
```
tail = newNode;
```



Initialization Insert C



Initialization Insert **D**



TestQueue.py

```
import sys
class Node :
    data = ""
    next = None
    prev = None

    def __init__ ( self, data):
        self.data = data

class Queue :
    __head = None
    __tail = None
    __size = 0

    def offer ( self, element):
        if self.__head == None :
            self.__head = Node( element)
            self.__tail = self.__head
        else :
            new_node = Node( element)
            new_node.next = self.__tail
            self.__tail.prev = new_node
            self.__tail = new_node
            self.__size+= 1

    def poll ( self):
        p = self.__head
        if p == None :
            return None
        self.__head = self.__head.prev
        p.next = None
        p.prev = None
        self.__size-= 1
        return p

@property
```

```

def size ( self ):
    return self. __size

def output ( self, queue):
    print ( "Head " , end= "" )
    node = queue. poll()
    while node != None :
        print ( node. data, " <- " , end= ""
    )
    node = queue. poll()
    print ( "Tail\n" )

def main ():
    queue = Queue()
    queue. offer( "A" )
    queue. offer( "B" )
    queue. offer( "C" )
    queue. offer( "D" )
    queue. output( queue)

if __name__ == "__main__" :
    main()

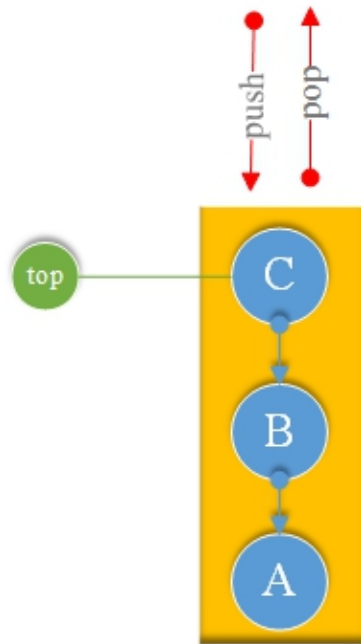
```

Result:

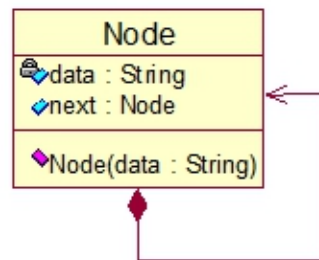
Stack

Stack:

FILO (First In Last Out) sequence.



UML Diagram



```
class Node :
```

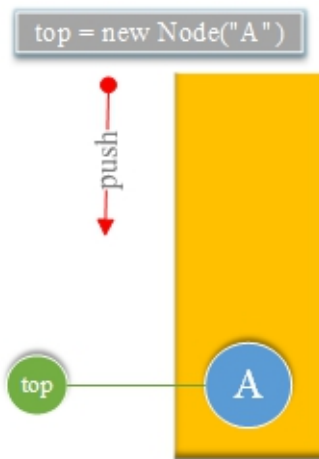
```
    data = "
```

```
    next = None
```

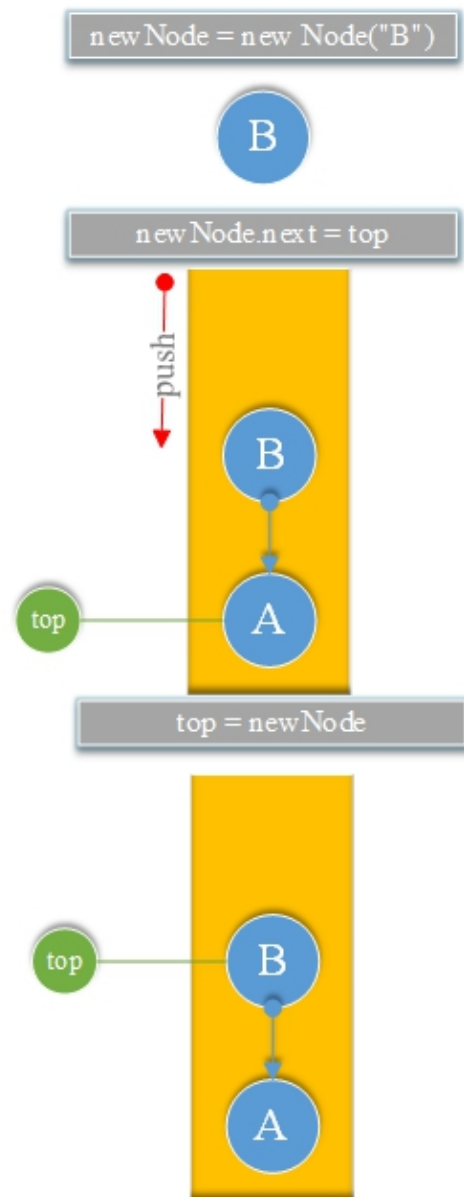
```
    def __init__ ( self, data):  
        self.data = data
```

1. Stack **initialization and traversal output** .

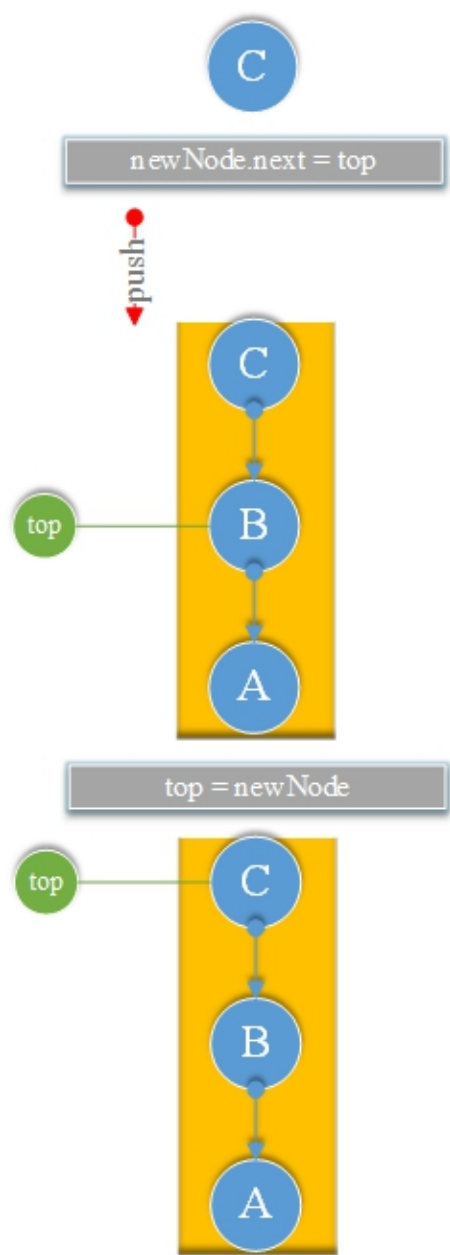
Push **A** into Stack



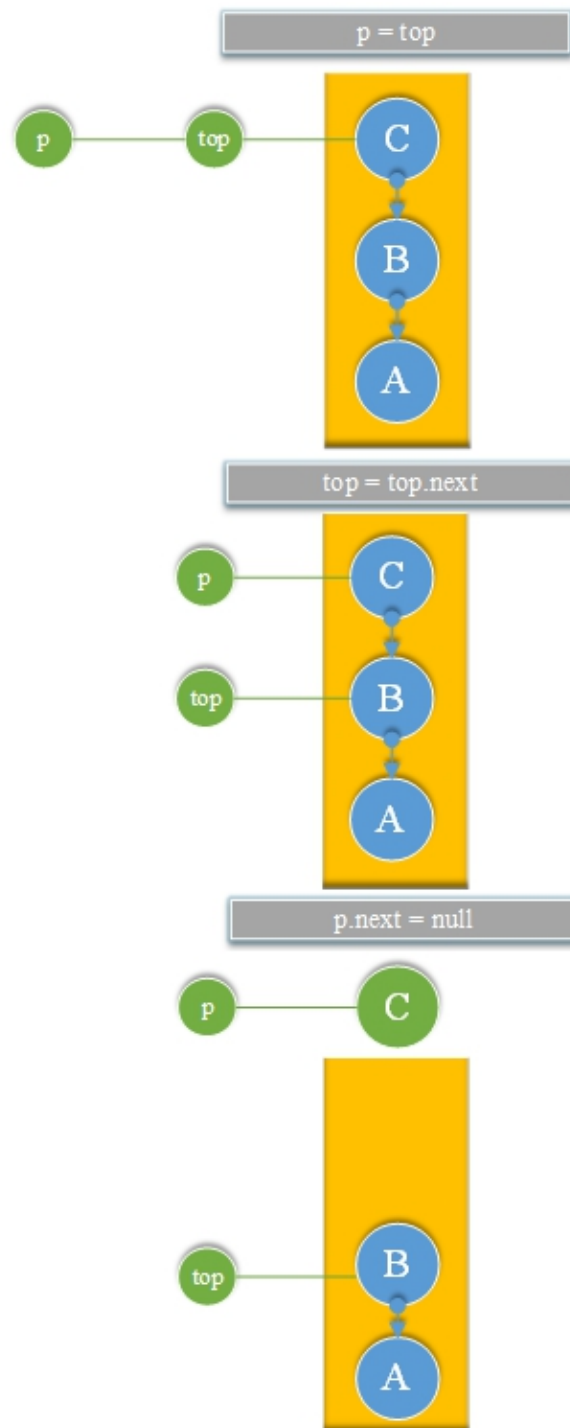
Push **B** into Stack



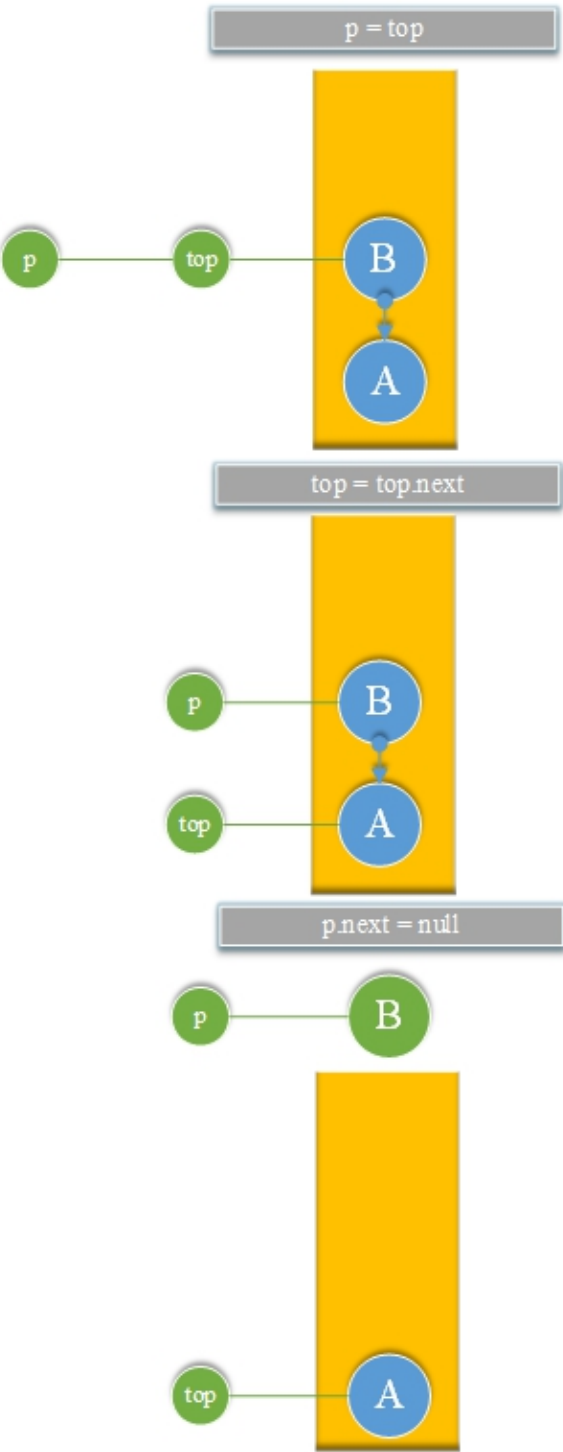
Push **C** into Stack



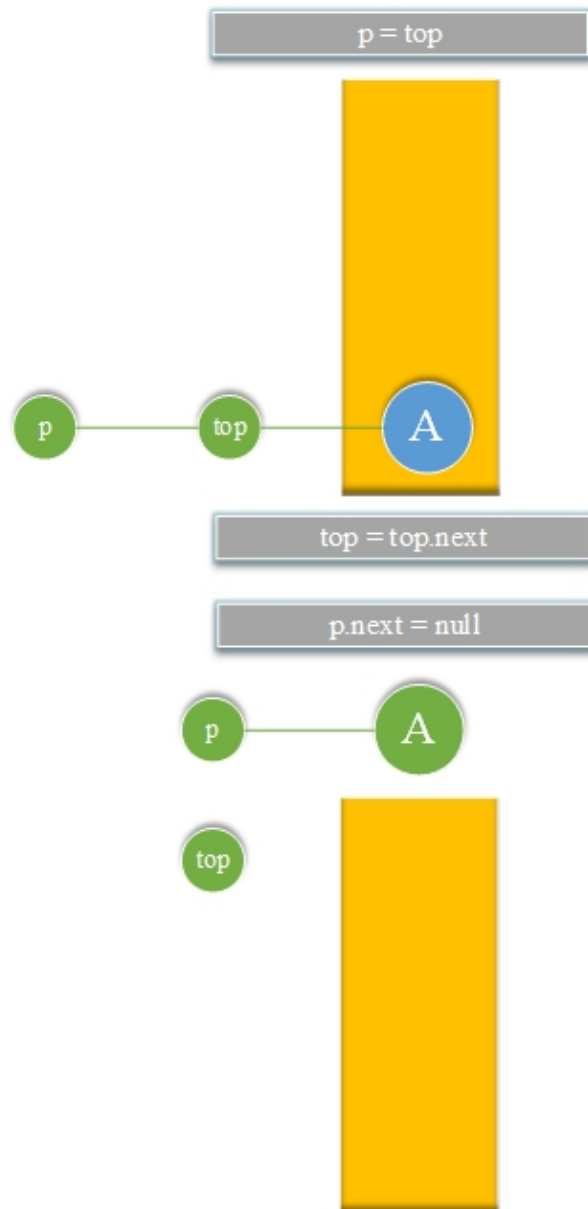
If pop **C** from Stack:



If pop **B** from Stack:



If pop **A** from Stack:



test_stack.py

```
import sys

class Node :
    data = ''
    next = None

    def __init__ ( self, data):
        self.data = data

class Stack :
    __top = None
    __size = 0

    def push ( self, element):
        if self.__top == None :
            self.__top = Node( element)
        else :
            new_node = Node( element)
            new_node.next = self.__top
            self.__top = new_node
            self.__size+= 1

    def pop ( self):
        if self.__top == None :
            return None
        p = self.__top
        self.__top = self.__top.next # top move down
        p.next = None
        self.__size-= 1
        return p

    @property
    def size ( self):
        return self.__size
```



```

def output ( self, stack ):
    print ( "Top " , end= "" )
    node = stack.pop()
    while node != None :
        print ( node.data, " -> " , end= "" )
        node = stack.pop()
    print ( "End\n" )

def main ():
    stack = Stack()
    stack.push( "A" )
    stack.push( "B" )
    stack.push( "C" )
    stack.push( "D" )
    stack.output( stack)

if __name__ == "__main__" :
    main()

```

Result:

Top D -> C -> B -> A -> End

Recursive Algorithm

Recursive Algorithm:

The program function itself calls its own layer to progress until it reaches a certain condition and step by step returns to the end..

1. Factorial of n : $n*(n-1)*(n-2) \dots *2*1$

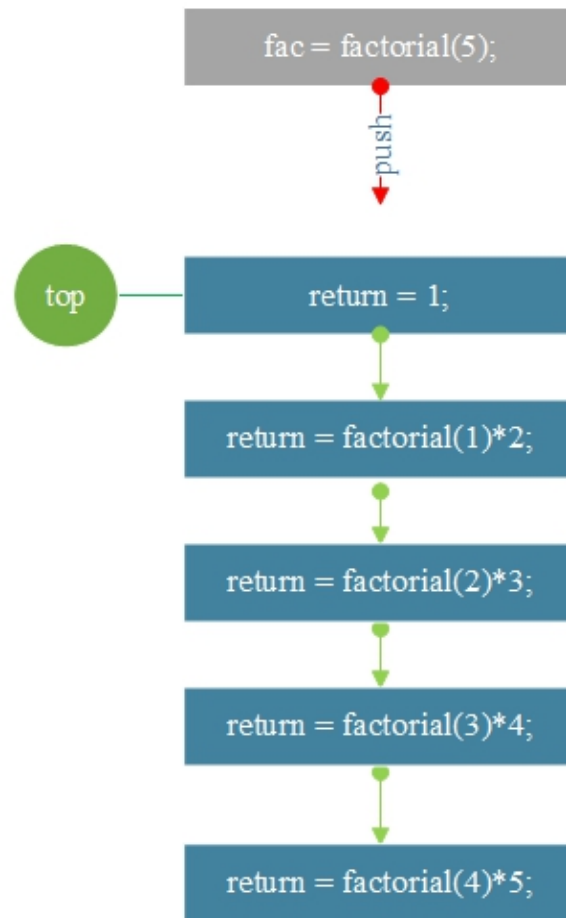
TestFactorial.py

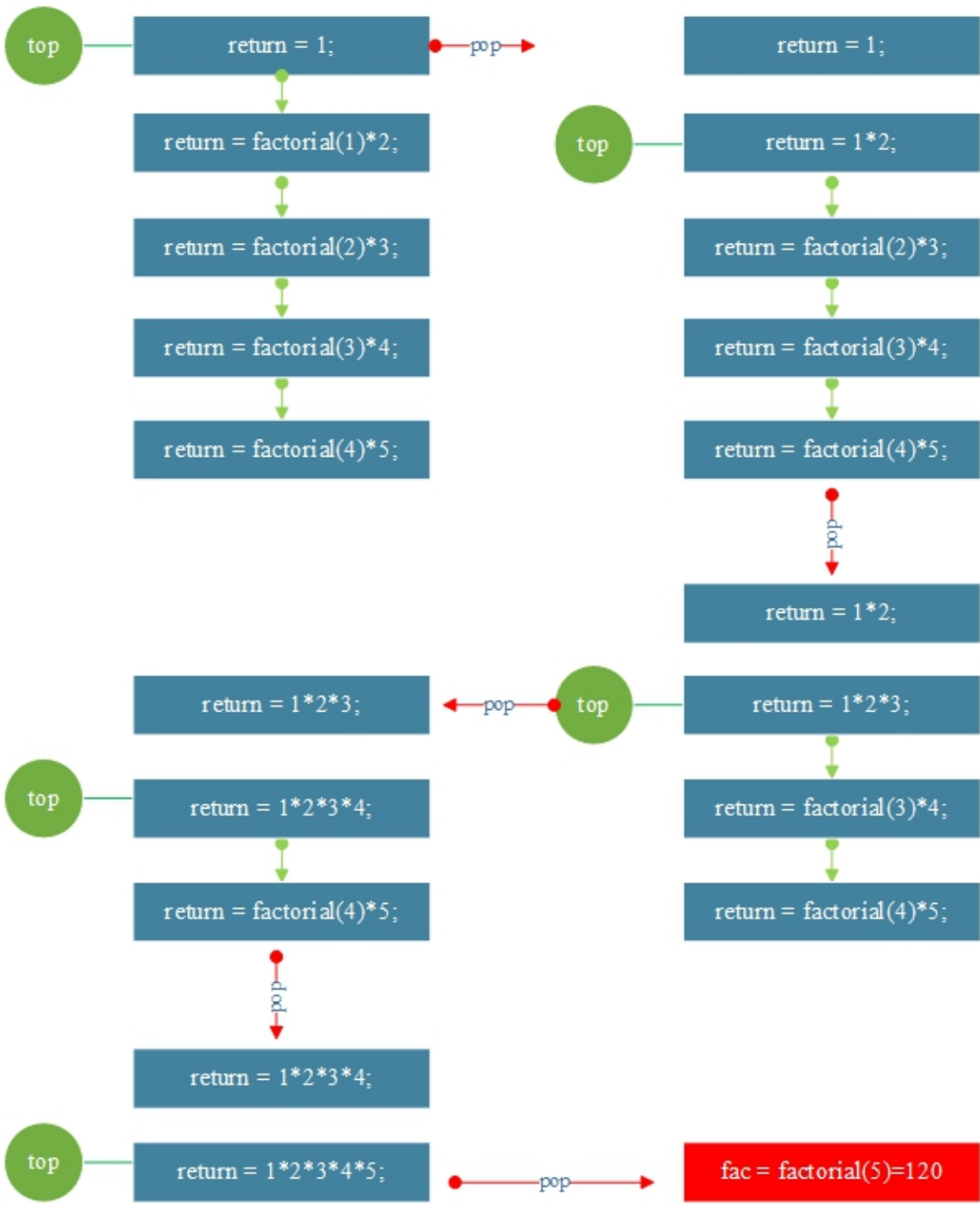
```
def factorial ( n):  
    if n == 1 :  
        return 1  
    else :  
        return factorial( n - 1 ) * n # Recursively call yourself until the  
end of the return  
  
def main ():  
    n = 5  
    fac = factorial( n)  
    print ( "The factorial of 5 is :", fac)  
  
if __name__ == "__main__" :  
    main()
```

Result:

The factorial of 5 is : 120

Graphical analysis:



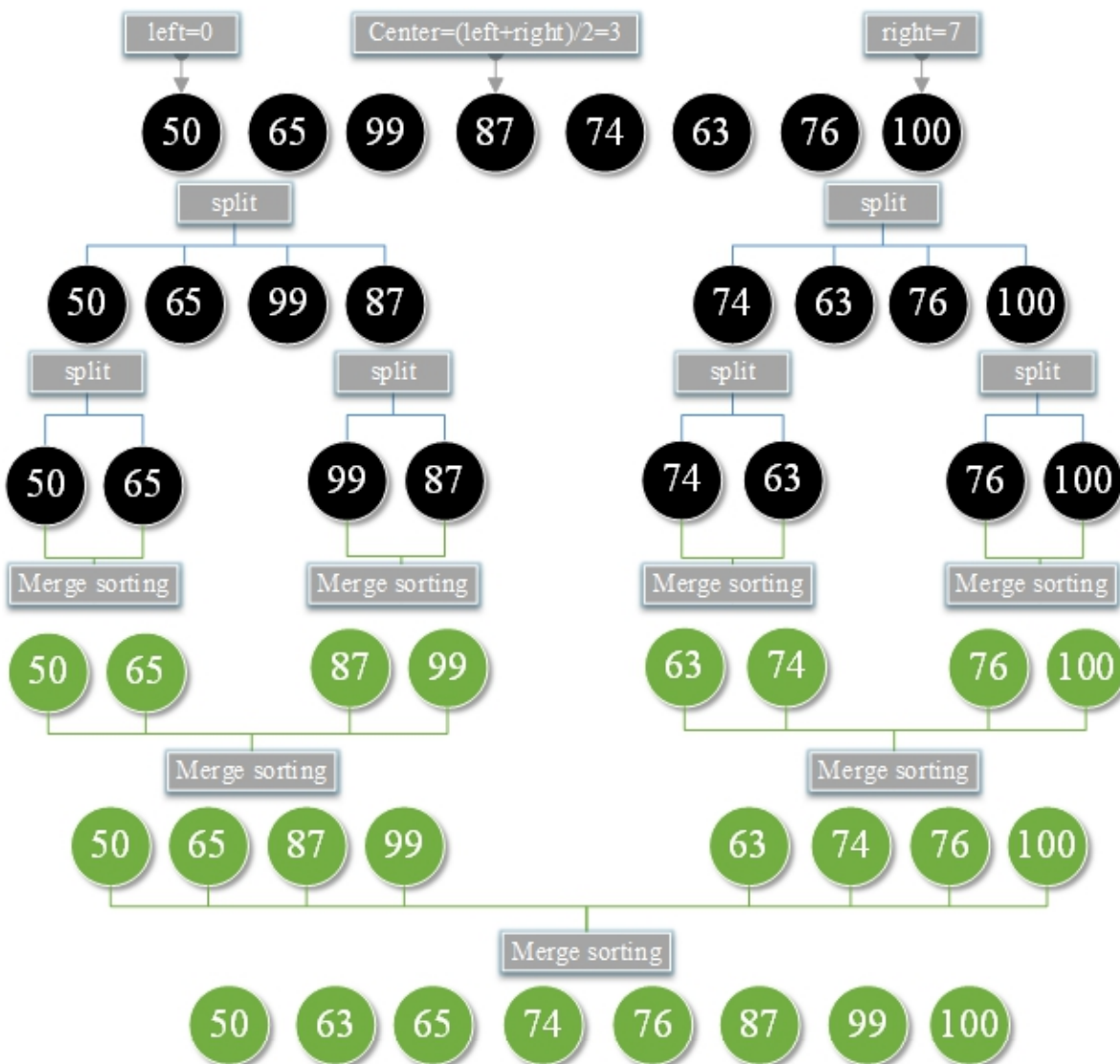


Two-way Merge Algorithm

Two-way Merge Algorithm:

The data of the first half and the second half are sorted, and the two ordered sub-list are merged into one ordered list, which continue to recursive to the end.

1. The scores {50, 65, 99, 87, 74, 63, 76, 100} by merge sort



test_merge_sort.py

```
def sort ( array):
    length = len( array)
    temp = [ 0 ]* length
    merge_sort( array, temp, 0 , length - 1 )

def merge_sort ( array, temp, left, right):
    if left < right:
        center = ( left + right) // 2
        merge_sort( array, temp, left, center) # Left merge sort
        merge_sort( array, temp, center + 1 , right) # Right merge sort
        merge( array, temp, left, center + 1 , right) # Merge two ordered
arrays

//Combine two ordered list into an ordered list
def merge ( array, temp, left, right, rightEndIndex):
    leftEndIndex = right - 1 #/ End subscript on the left
    tempIndex = left #Starting from the left count
    elementNumber = rightEndIndex - left + 1
    while left <= leftEndIndex and right <= rightEndIndex:
        if array[ left] <= array[ right]:
            temp[ tempIndex] = array[ left]
            tempIndex+= 1
            left+= 1
        else :
            temp[ tempIndex] = array[ right]
            tempIndex+= 1
            right+= 1

    while left <= leftEndIndex: # If there is element on the left
        temp[ tempIndex] = array[ left]
        tempIndex+= 1
        left+= 1

    while right <= rightEndIndex: #If there is element on the right
        temp[ tempIndex] = array[ right]
        tempIndex+= 1
```

```
        right+= 1

    for i in range( 0 , elementNumber):
        array[ rightEndIndex] = temp[ rightEndIndex]
        rightEndIndex-= 1

def main ():
    scores = [ 50 , 65 , 99 , 87 , 74 , 63 , 76 , 100 , 92 ]
    sort( scores)
    for score in scores:
        print ( score, ", " , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

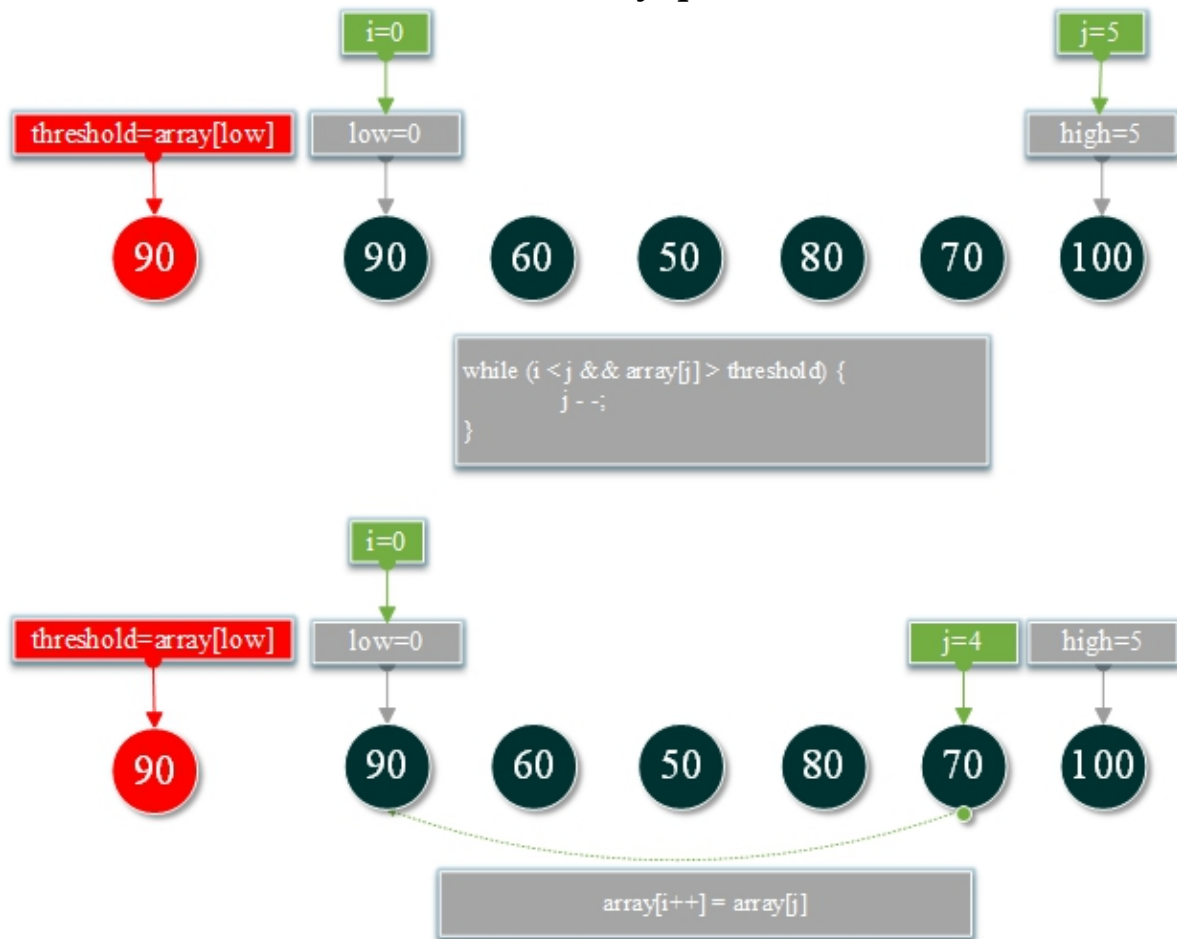
50 ,63 ,65 ,74 ,76 ,87 ,92 ,99 ,100 ,

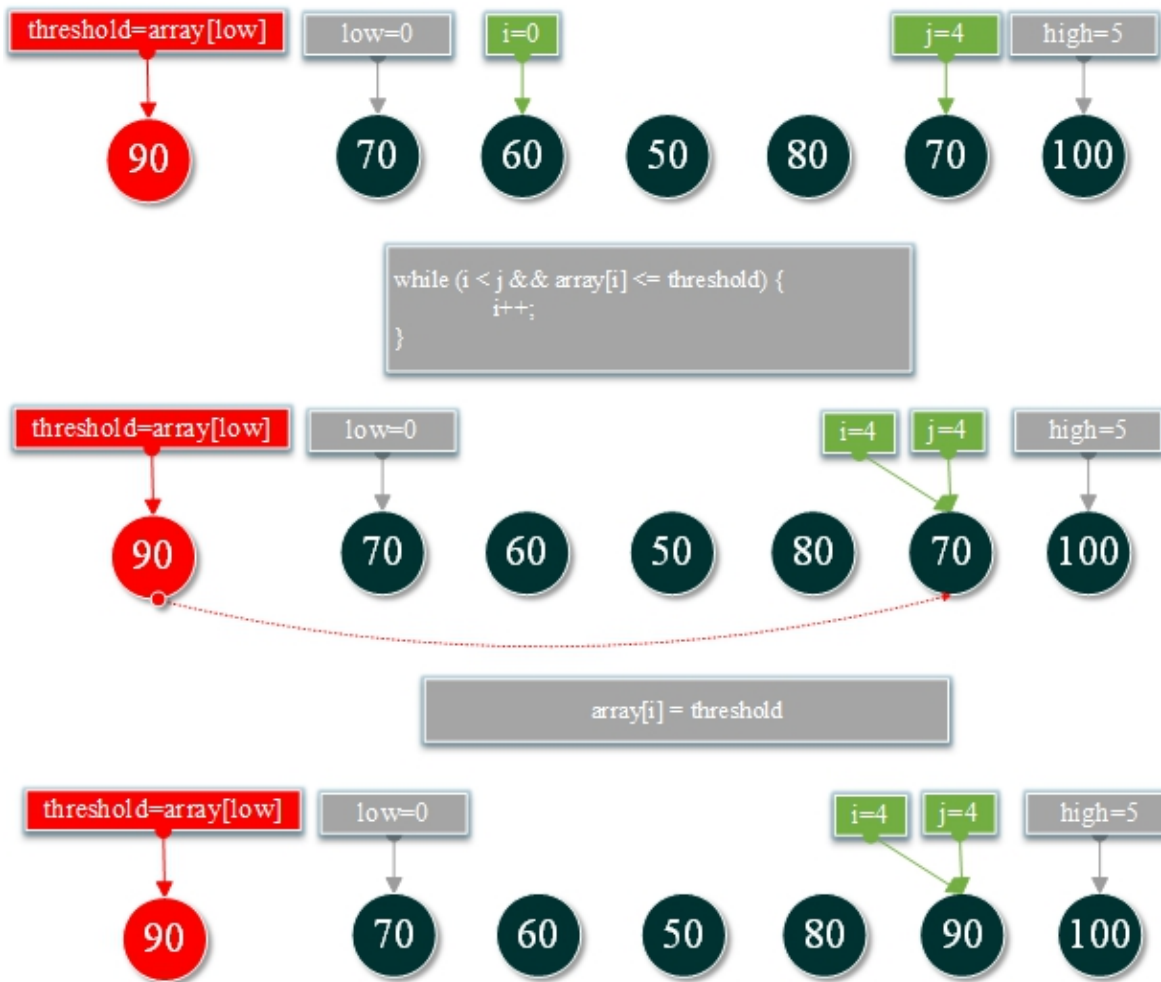
Quick Sort Algorithm

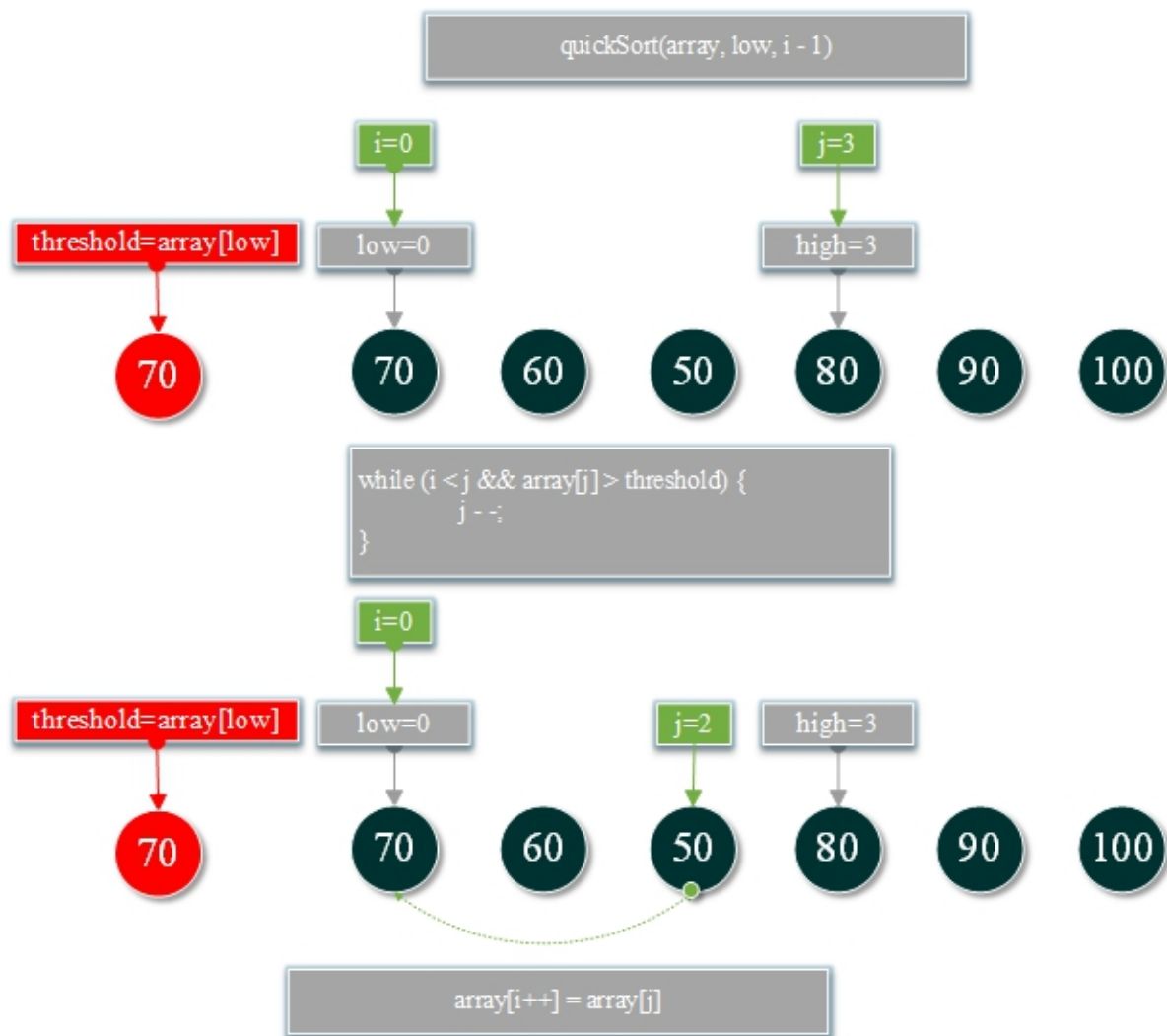
Quick Sort Algorithm:

Quicksort is a popular sorting algorithm that is often faster in practice compared to other sorting algorithms. It utilizes a divide-and-conquer strategy to quickly sort data items by dividing a large array into two smaller arrays.

1. The scores {90, 60, 50, 80, 70, 100} by quick sort









test_quick_sort.py

```
def sort ( array):
    length = len( array)
    if length > 0 :
        quick_sort( array, 0 , length - 1 )

def quick_sort ( array, low, high):
    if low > high :
        return

    i = low
    j = high
    threshold = array[ low]

    # Alternately scanned from both ends of the list
    while i < j:
        # Find the first position less than threshold from right
        # to left
        while i < j and array[ j] > threshold:
            j -= 1

        # Replace the low with a smaller number than the
        # threshold
        if i < j:
            array[ i] = array[ j]
            i += 1

        # Find the first position greater than threshold from
        # left to right
        while i < j and array[ i] <= threshold:
            i += 1

        # Replace the high with a number larger than the
        # threshold
        if i < j:
            array[ j] = array[ i]
            j -= 1
```

```

    array[ i] = threshold

    # left quick_sort
    quick_sort( array, low, i - 1 )
    # right quick_sort
    quick_sort( array, i + 1 , high)

def main ():
    scores = [ 90 , 60 , 50 , 80 , 70 , 100 ]
    sort( scores)
    for score in scores:
        print ( score, " , " , end= "" )

if __name__ == "__main__" :
    main()

```

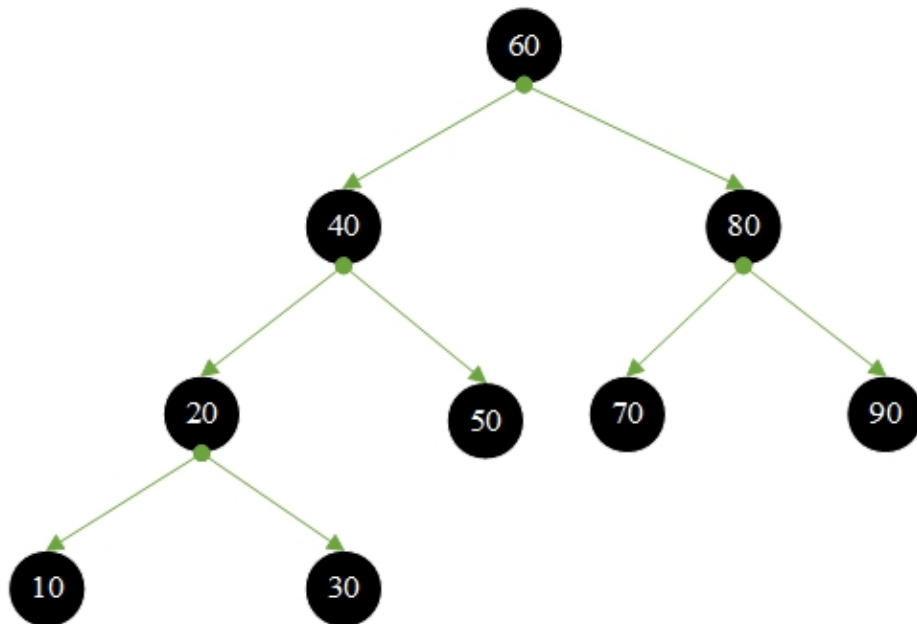
Result:

50 ,60 ,70 ,80 ,90 ,100 ,

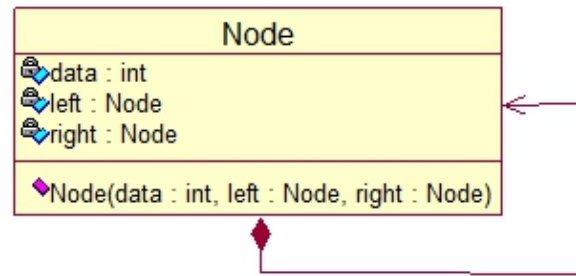
Binary Search Tree

Binary Search Tree:

1. If the left subtree of any node is not empty, the value of all nodes on the left subtree is less than the value of its root node;
2. If the right subtree of any node is not empty, the value of all nodes on the right subtree is greater than the value of its root node;
3. The left subtree and the right subtree of any node are also binary search trees.



Node UML Diagram



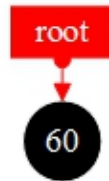
```
class Node :
    data = ""
    left = None
    right = None

    def __init__( self, data, left, right):
        self.data = data;
        self.left = left;
        self.right = right;
```

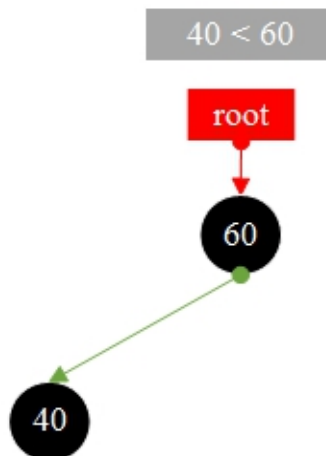
1. Construct a binary search tree, insert node

The inserted nodes are compared from the root node, and the smaller than the root node is compared with the left subtree of the root node, otherwise, compared with the right subtree until the left subtree is empty or the right subtree is empty, then is inserted.

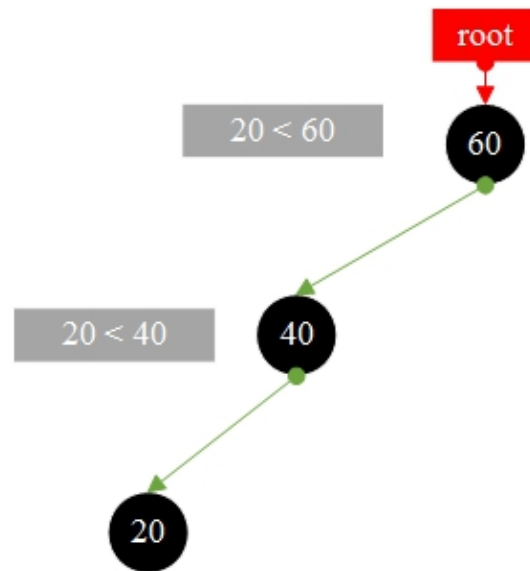
Insert 60



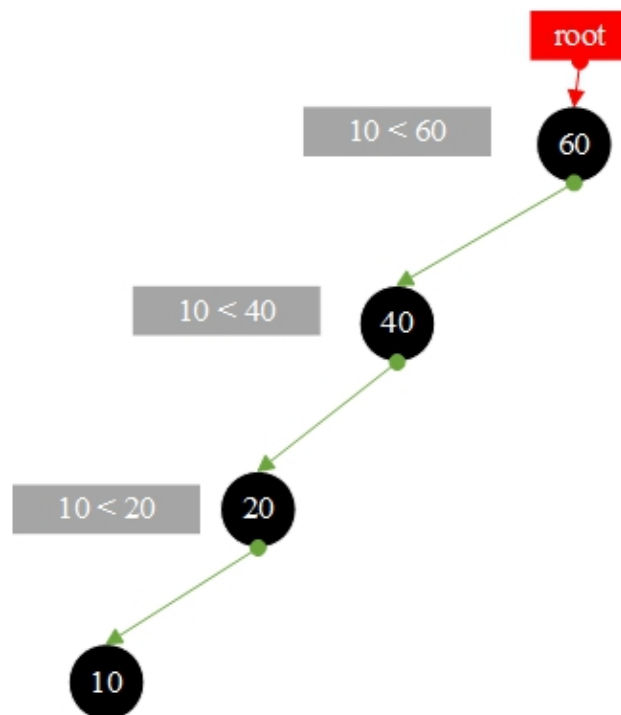
Insert 40



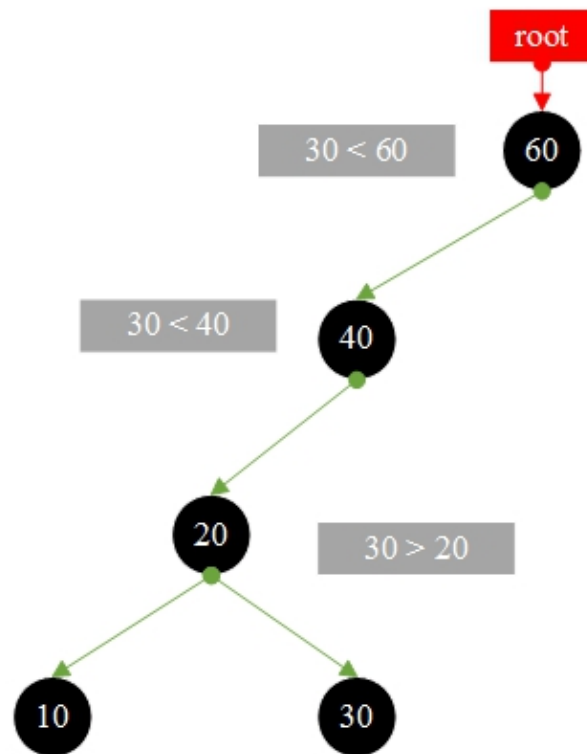
Insert **20**



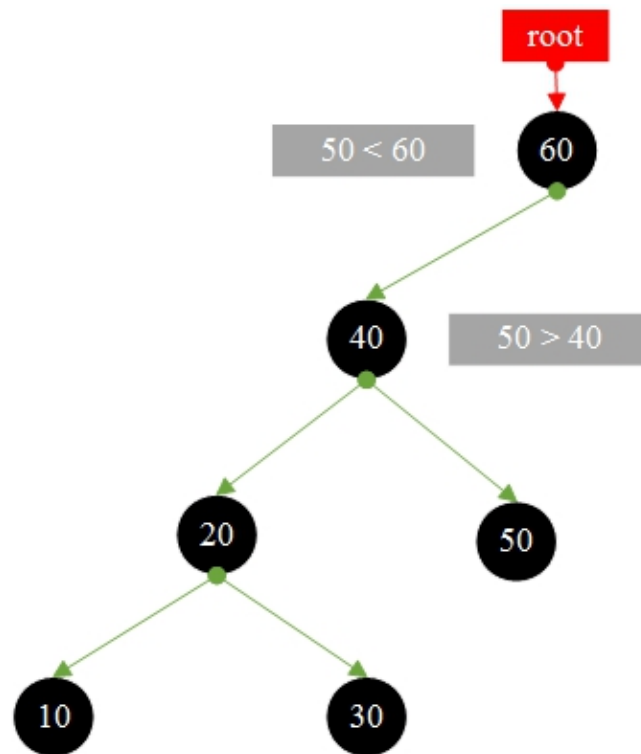
Insert **10**



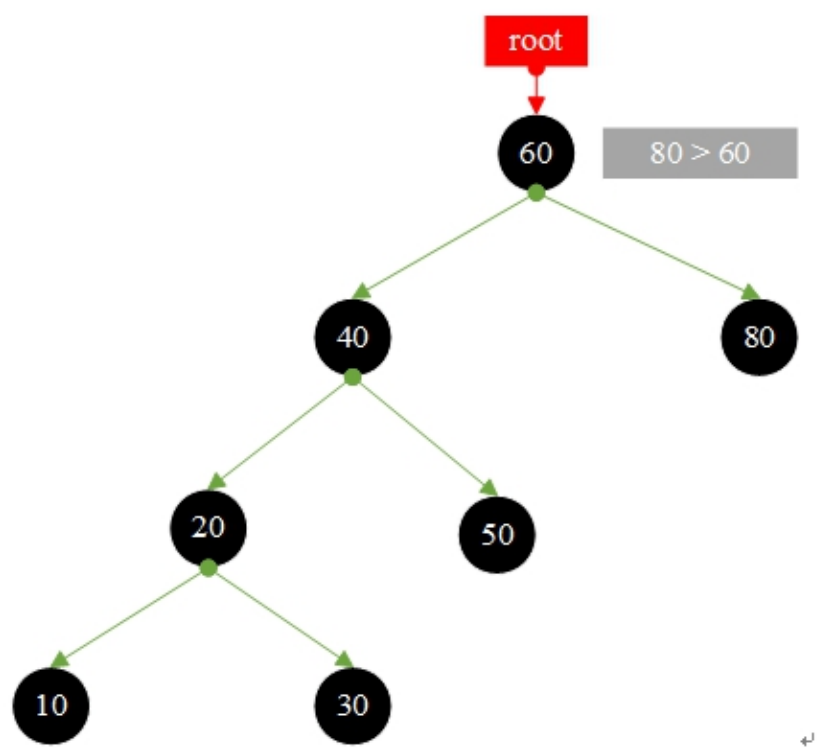
Insert **30**



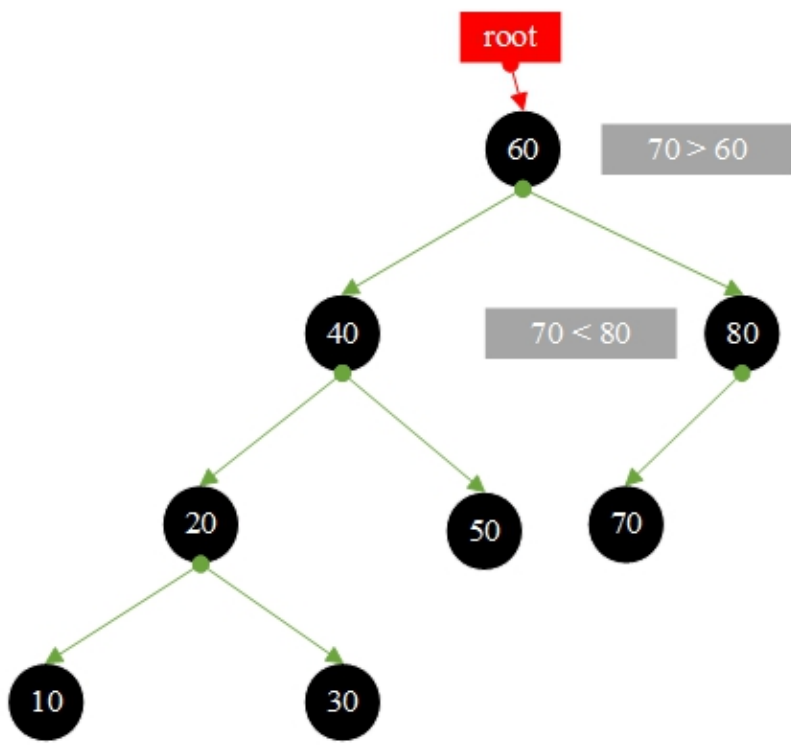
Insert **50**



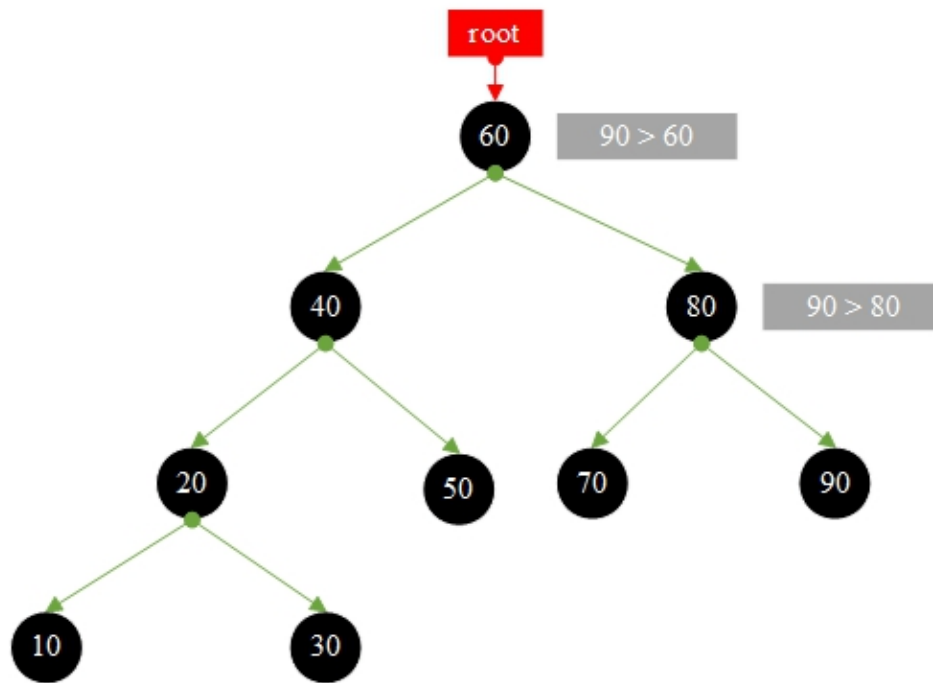
Insert **80**



Insert **70**

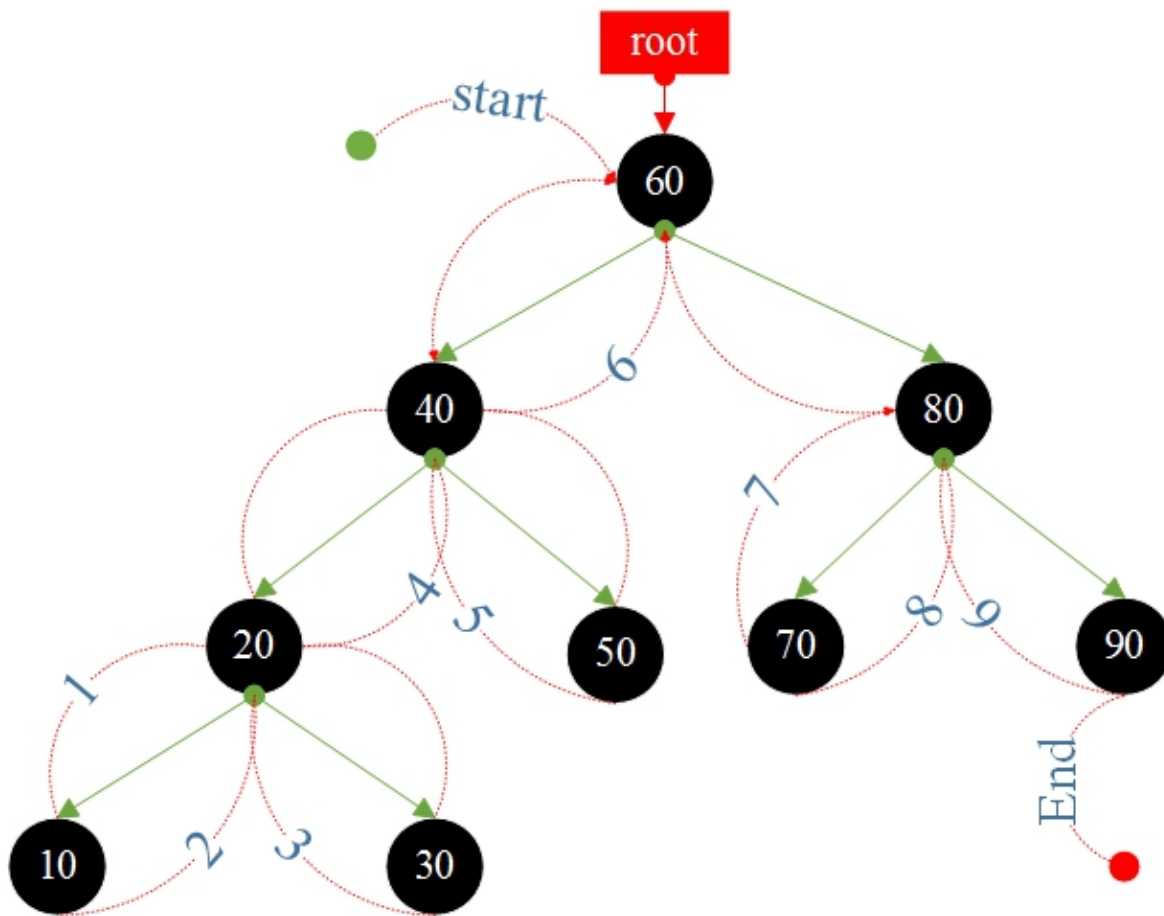


Insert **90**

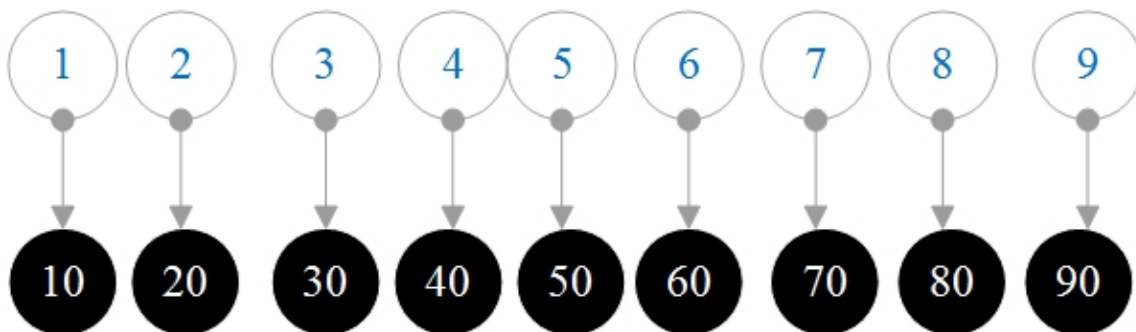


2. binary search tree **In-order traversal**

In-order traversal : left subtree -> root node -> right subtree



Result:



test_binary_tree.py

```
class Node :
    data = ""
    left = None
    right = None

    def __init__ ( self, data, left, right):
        self.data = data;
        self.left = left;
        self.right = right;

class BinaryTree :
    __root = None

    @property
    def root ( self):
        return self.__root

    def inOrder ( self, root):
        if root == None :
            return
        self.inOrder( root.left) # Traversing the left subtree
        print ( root.data, " , " , end= "" )
        self.inOrder( root.right) # Traversing the right subtree

    def insert ( self, node, new_data):
        if self.__root == None :
            self.__root = Node( new_data, None , None )
            return
        compareValue = new_data - node.data
        #Recursive left subtree, continue to find the insertion position
        if compareValue < 0 :
            if node.left == None :
                node.left = Node( new_data, None , None )
            else :
                self.insert( node.left, new_data)
```



```

        elif compareValue > 0 : #Recursive right subtree, continue to find
the insertion position
            if node. right == None :
                node. right = Node( new_data, None , None )
            else :
                self. insert( node. right, new_data)

def main ():
    binary_tree = BinaryTree()
    #Constructing a binary search tree
    binary_tree. insert( binary_tree. root, 60 )
    binary_tree. insert( binary_tree. root, 40 )
    binary_tree. insert( binary_tree. root, 20 )
    binary_tree. insert( binary_tree. root, 10 )
    binary_tree. insert( binary_tree. root, 30 )
    binary_tree. insert( binary_tree. root, 50 )
    binary_tree. insert( binary_tree. root, 80 )
    binary_tree. insert( binary_tree. root, 70 )
    binary_tree. insert( binary_tree. root, 90 )

    print ( "\nIn-order traversal binary search tree" )
    binary_tree. inOrder( binary_tree. root)

if __name__ == "__main__" :
    main()

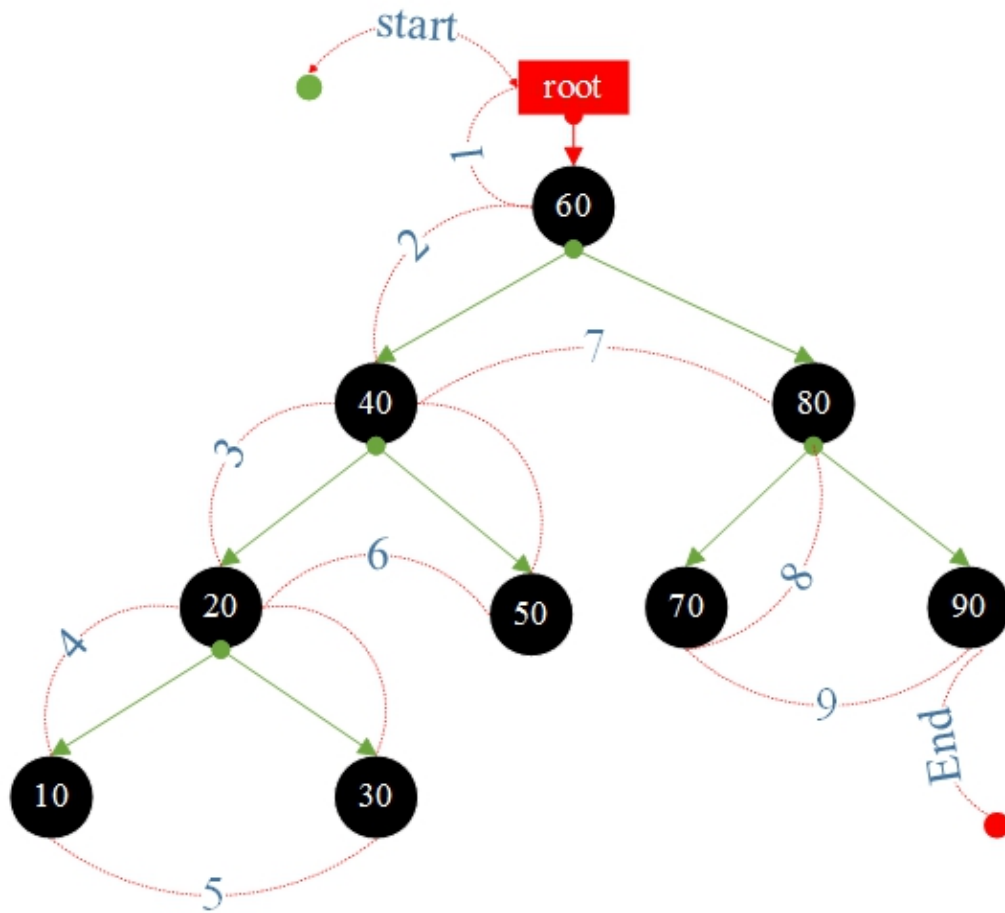
```

Result:

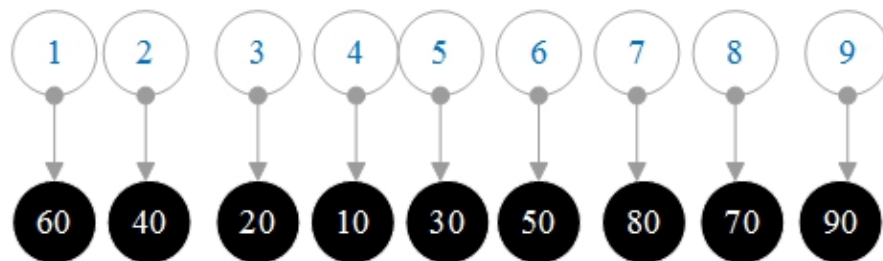
In-order traversal binary search tree
10 , 20 , 30 , 40 , 50 , 60 , 70 , 80 , 90 ,

3. binary search tree **Pre-order traversal**

Pre-order traversal : root node -> left subtree -> right subtree



Result:



test_binary_tree.py

```
class Node :
    data = ""
    left = None
    right = None

    def __init__ ( self, data, left, right):
        self.data = data;
        self.left = left;
        self.right = right;

class BinaryTree :
    __root = None

    @property
    def root ( self):
        return self.__root

    def preOrder ( self, root):
        if root == None :
            return
        print ( root.data, ", ", end= "" )
        self.preOrder( root.left) # Recursive Traversing the left subtree
        self.preOrder( root.right) # Recursive Traversing the right subtree

    def insert ( self, node, new_data):
        if self.__root == None :
            self.__root = Node( new_data, None , None )
            return
        compareValue = new_data - node.data
        #Recursive left subtree, continue to find the insertion position
        if compareValue < 0 :
            if node.left == None :
                node.left = Node( new_data, None , None )
            else :
                self.insert( node.left, new_data)
```

```

        elif compareValue > 0 : #Recursive right subtree, continue to find
the insertion position
            if node. right == None :
                node. right = Node( new_data, None , None )
            else :
                self. insert( node. right, new_data)

def main ():
    binary_tree = BinaryTree()
    #Constructing a binary search tree
    binary_tree. insert( binary_tree. root, 60 )
    binary_tree. insert( binary_tree. root, 40 )
    binary_tree. insert( binary_tree. root, 20 )
    binary_tree. insert( binary_tree. root, 10 )
    binary_tree. insert( binary_tree. root, 30 )
    binary_tree. insert( binary_tree. root, 50 )
    binary_tree. insert( binary_tree. root, 80 )
    binary_tree. insert( binary_tree. root, 70 )
    binary_tree. insert( binary_tree. root, 90 )

    print ( "Pre-order traversal binary search tree" )
    binary_tree. preOrder( binary_tree. root)

if __name__ == "__main__" :
    main()

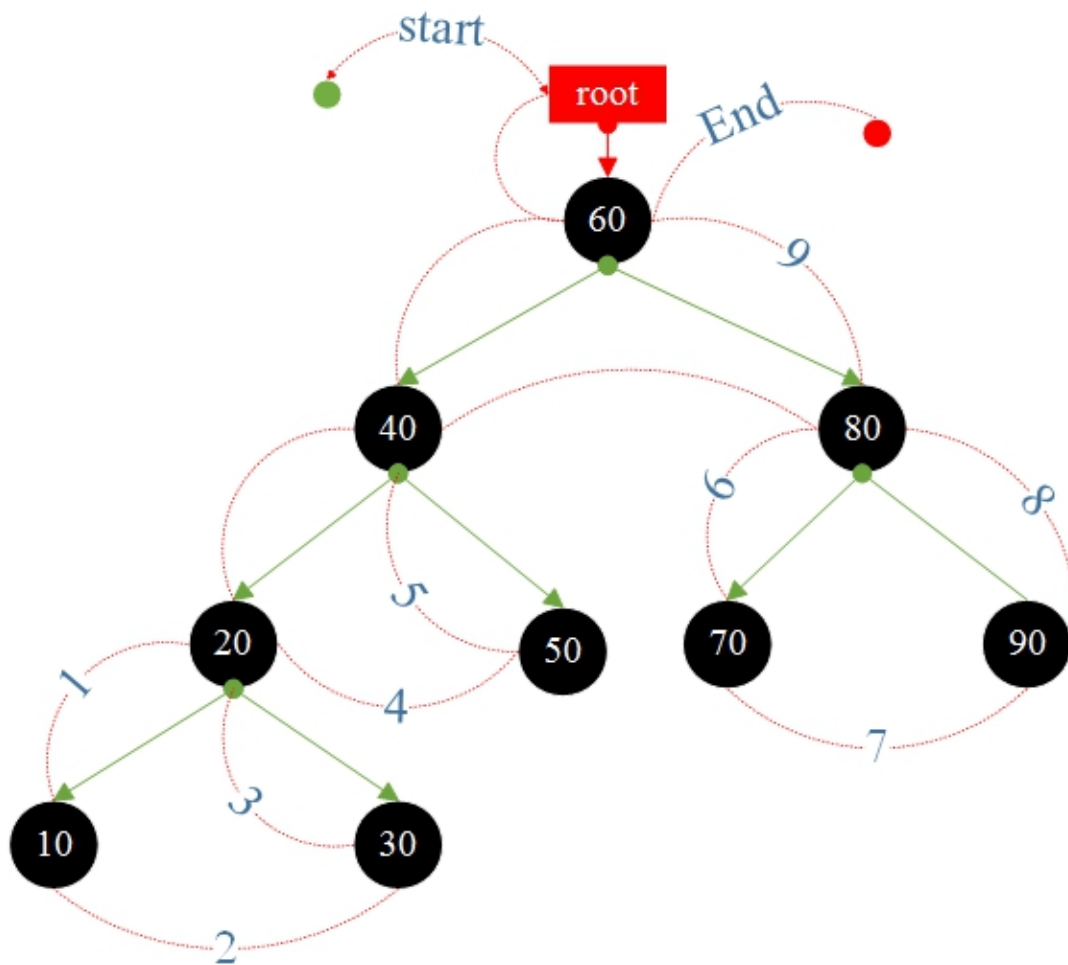
```

Result:

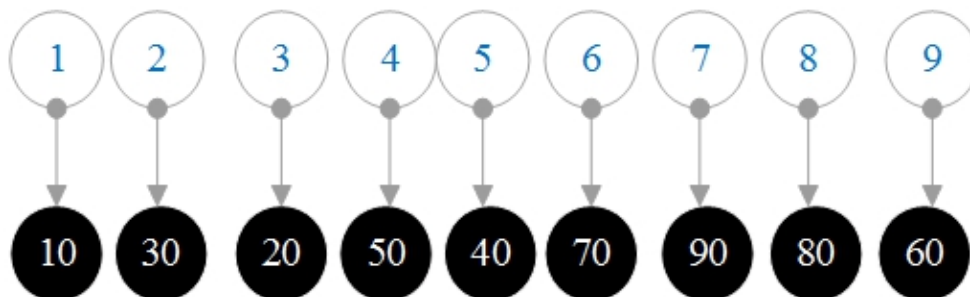
Pre-order traversal binary search tree
60 , 40 , 20 , 10 , 30 , 50 , 80 , 70 , 90 ,

4. binary search tree **Post-order traversal**

Post-order traversal : right subtree -> root node -> left subtree



Result:



test_binary_tree.py

```
class Node :
    data = ""
    left = None
    right = None

    def __init__ ( self, data, left, right):
        self.data = data;
        self.left = left;
        self.right = right;

class BinaryTree :
    __root = None

    @property
    def root ( self):
        return self.__root

    def postOrder ( self, root):
        if root == None :
            return
        self.postOrder( root.left) # Recursive Traversing the left subtree
        self.postOrder( root.right) # Recursive Traversing the right
subtree
        print ( root.data, ", " , end= "" )

    def insert ( self, node, new_data):
        if self.__root == None :
            self.__root = Node( new_data, None , None )
            return
        compareValue = new_data - node.data
        #Recursive left subtree, continue to find the insertion position
        if compareValue < 0 :
            if node.left == None :
                node.left = Node( new_data, None , None )
            else :
                self.insert( node.left, new_data)
```

```

        elif compareValue > 0 : #Recursive right subtree, continue to find
the insertion position
            if node. right == None :
                node. right = Node( new_data, None , None )
            else :
                self. insert( node. right, new_data)

def main ():
    binary_tree = BinaryTree()
    #Constructing a binary search tree
    binary_tree. insert( binary_tree. root, 60 )
    binary_tree. insert( binary_tree. root, 40 )
    binary_tree. insert( binary_tree. root, 20 )
    binary_tree. insert( binary_tree. root, 10 )
    binary_tree. insert( binary_tree. root, 30 )
    binary_tree. insert( binary_tree. root, 50 )
    binary_tree. insert( binary_tree. root, 80 )
    binary_tree. insert( binary_tree. root, 70 )
    binary_tree. insert( binary_tree. root, 90 )

    print ( "Post-order traversal binary search tree" )
    binary_tree. postOrder( binary_tree. root)

if __name__ == "__main__" :
    main()

```

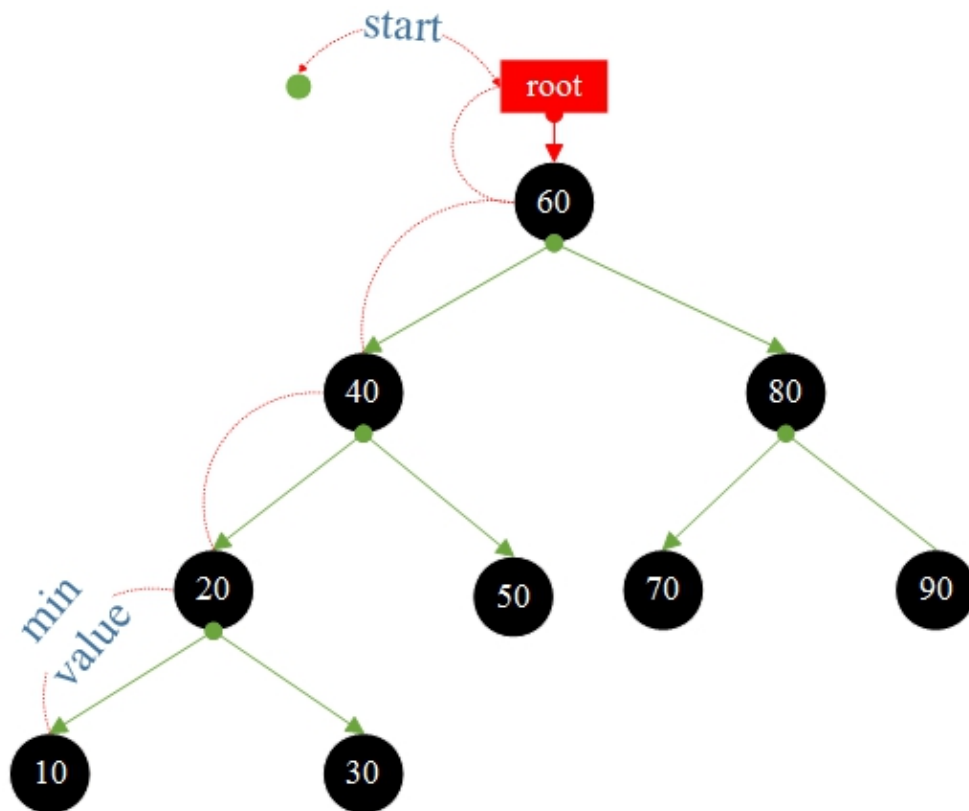
Result:

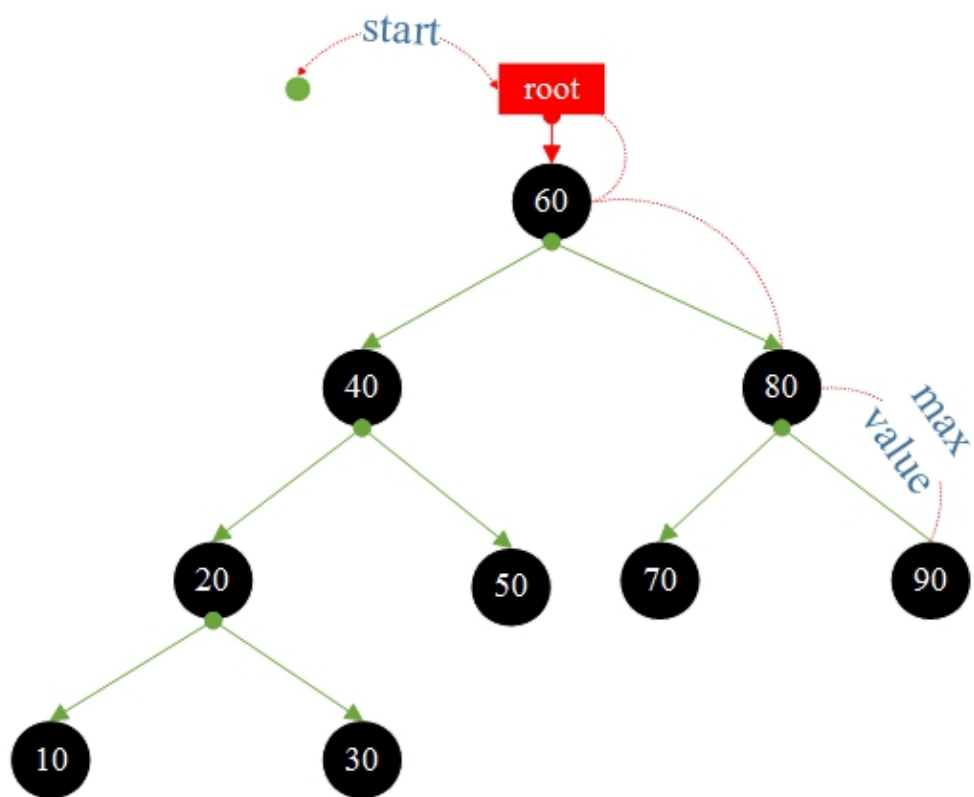
Post-order traversal binary search tree
10 , 30 , 20 , 50 , 40 , 70 , 90 , 80 , 60 ,

5. binary search tree **Maximum and minimum**

Minimum value: The small value is on the left child node, as long as the recursion traverses the left child until be empty, the current node is the minimum node.

Maximum value: The large value is on the right child node, as long as the recursive traversal is the right child until be empty, the current node is the largest node.





test_binary_tree.py

```
class Node :
    data = ""
    left = None
    right = None

    def __init__ ( self, data, left, right):
        self.data = data;
        self.left = left;
        self.right = right;

class BinaryTree :
    __root = None

    @property
    def root ( self):
        return self.__root

    #Minimum value
    def search_minimum_value ( self, node):
        if node == None or node.data == 0 :
            return None
        if node.left == None :
            return node
        # Recursively find the minimum from the left subtree
        return self.search_minimum_value( node.left)

    #Maximum value
    def search_maximum_value ( self, node):
        if node == None or node.data == 0 :
            return None
        if node.right == None :
            return node
        # Recursively find the minimum from the right subtree
        return self.search_maximum_value( node.right)
```

```

def insert ( self, node, new_data ):
    if self. __root == None :
        self. __root = Node( new_data, None , None )
        return
    compareValue = new_data - node. data
    #Recursive left subtree, continue to find the insertion position
    if compareValue < 0 :
        if node. left == None :
            node. left = Node( new_data, None , None )
        else :
            self. insert( node. left, new_data)
    elif compareValue > 0 : #Recursive right subtree, continue to find
the insertion position
        if node. right == None :
            node. right = Node( new_data, None , None )
        else :
            self. insert( node. right, new_data)

def main ():
    binary_tree = BinaryTree()
    binary_tree. insert( binary_tree. root, 60 )
    binary_tree. insert( binary_tree. root, 40 )
    binary_tree. insert( binary_tree. root, 20 )
    binary_tree. insert( binary_tree. root, 10 )
    binary_tree. insert( binary_tree. root, 30 )
    binary_tree. insert( binary_tree. root, 50 )
    binary_tree. insert( binary_tree. root, 80 )
    binary_tree. insert( binary_tree. root, 70 )
    binary_tree. insert( binary_tree. root, 90 )

    print ( "\nMinimum Value" )
    min_node= binary_tree. search_minimum_value( binary_tree. root)
    print ( min_node. data)
    print ( "\nMaximum Value" )
    max_node= binary_tree. search_maximum_value( binary_tree. root)
    print ( max_node. data)

if __name__ == "__main__" :

```

```
main()
```

Result:

Minimum Value

10

Maximum Value

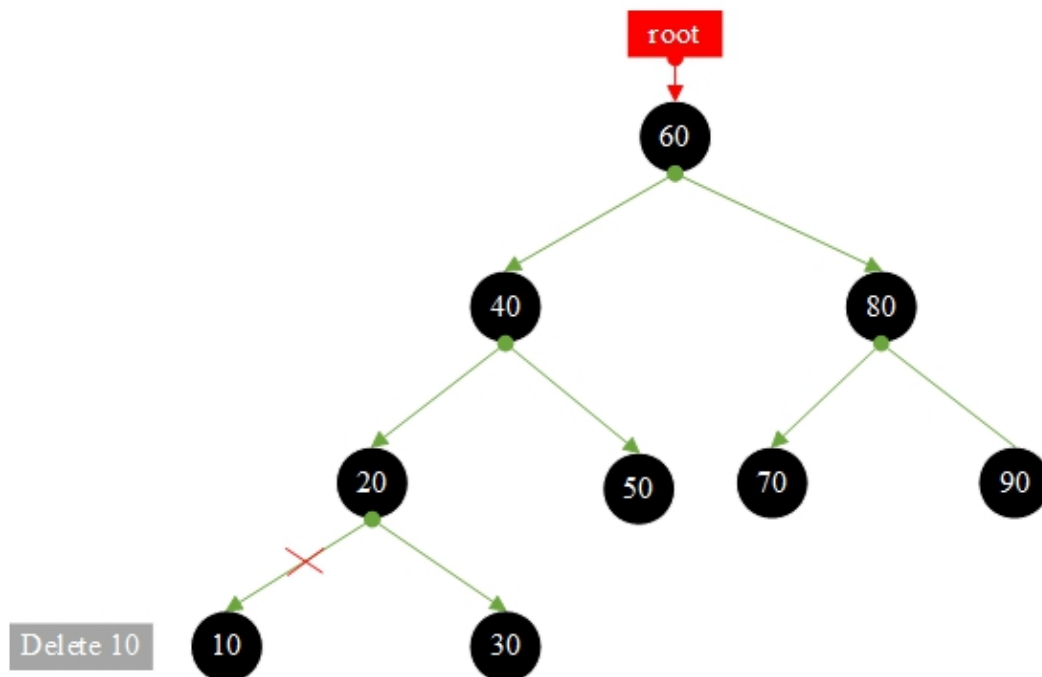
90

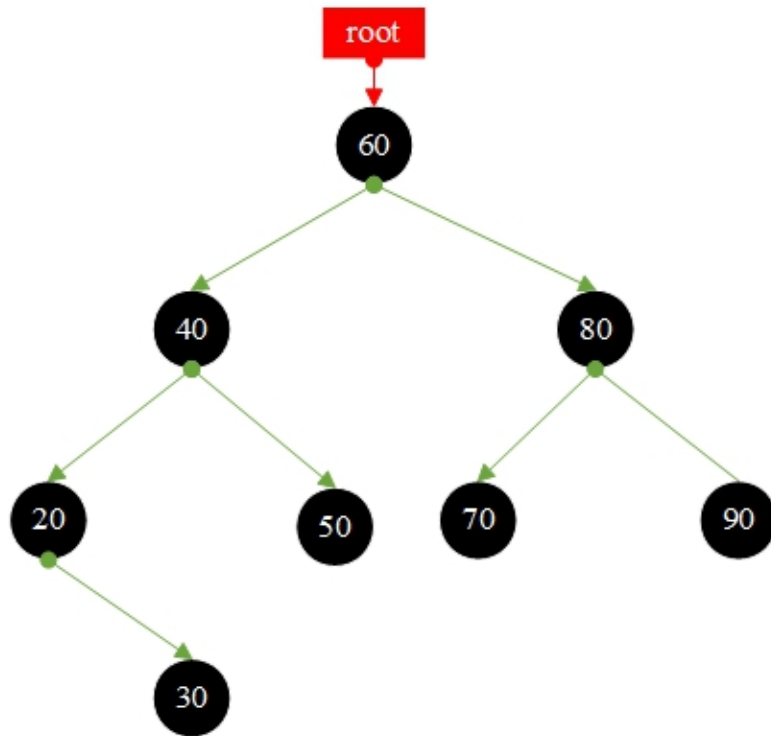
6. binary search tree **Delete Node**

Binary search tree delete node 3 cases

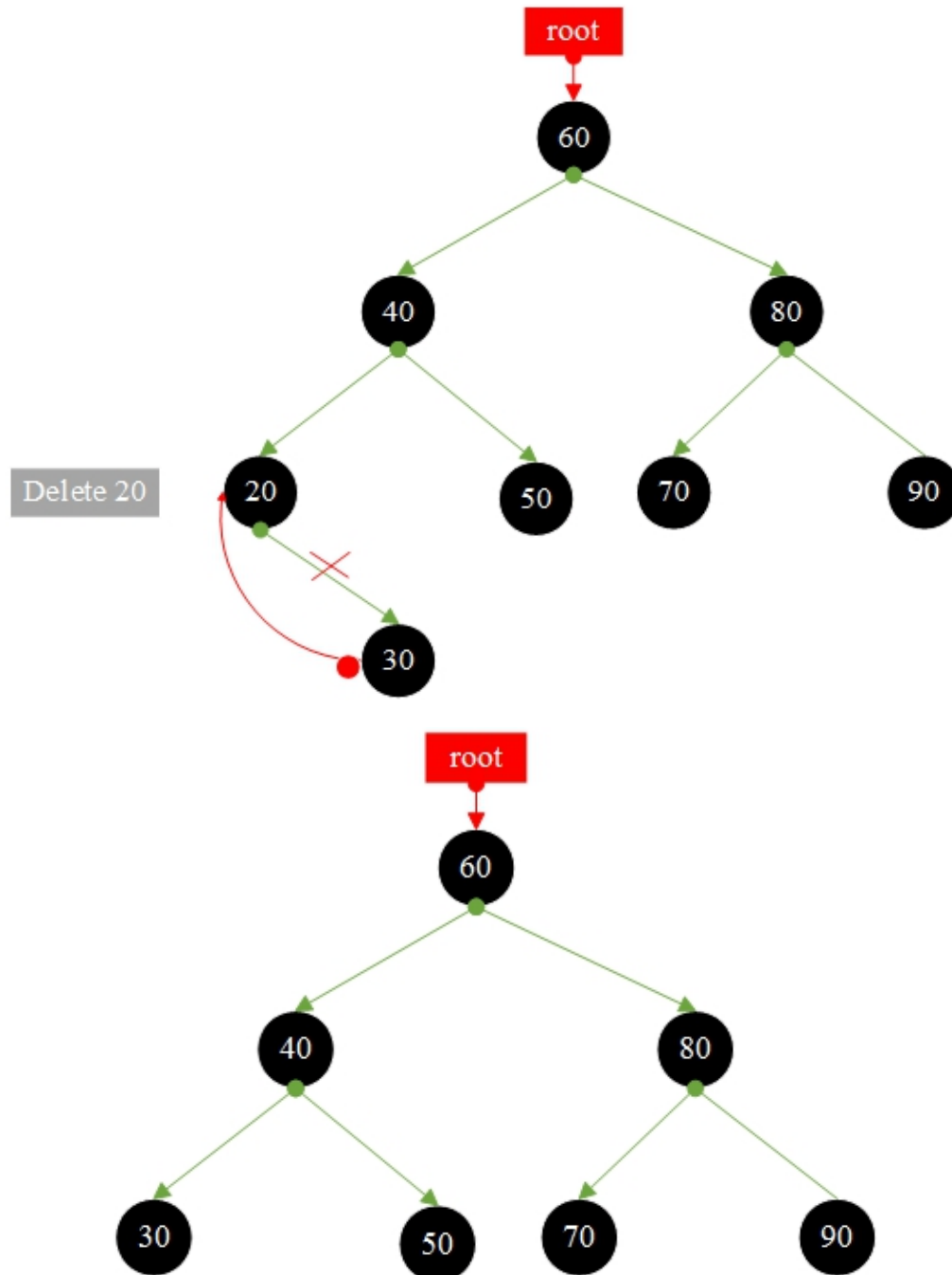
1. If there is no child node, delete it directly
2. If there is only one child node, the child node replaces the current node, and then deletes the current node.
3. If there are two child nodes, replace the current node with the smallest node from the right subtree, because the smallest node on the right is also larger than the value on the left.

1. If there is no child node, delete it directly: delete node 10

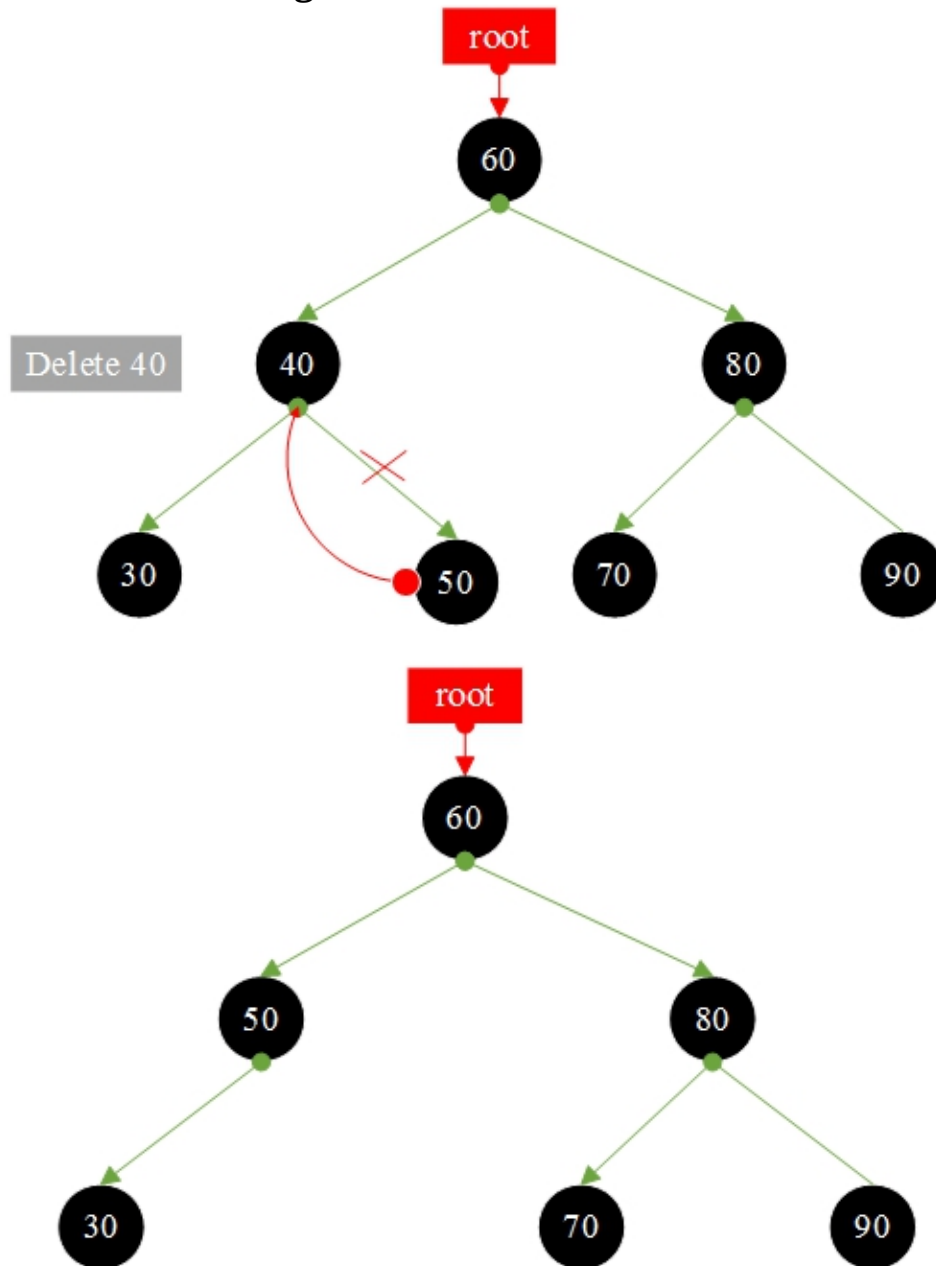




2. If there is only one child node, the child node replaces the current node, and then deletes the current node. **Delete node 20**



3. If there are two child nodes, replace the current node with the smallest node from the right subtree, **Delete node 40**



test_binary_tree.py

```
class Node :
    data = ""
    left = None
    right = None

    def __init__ ( self, data, left, right):
        self.data = data;
        self.left = left;
        self.right = right;

class BinaryTree :
    __root = None

    @property
    def root ( self):
        return self.__root

    def inOrder ( self, root):
        if root == None :
            return
        self.inOrder( root.left) # Traversing the left subtree
        print ( root.data, " , " , end= "" )
        self.inOrder( root.right) # Traversing the right subtree

    def remove ( self, node, new_data):
        if node == None :
            return node
        compareValue = new_data - node.data
        if compareValue > 0 :
            node.right = self.remove( node.right, new_data)
        elif compareValue < 0 :
            node.left = self.remove( node.left, new_data)
        elif node.left != None and node.right != None :
            # Find the minimum node of the right subtree to replace the
current node
            node.data = self.search_minimum_value( node.right).data
            node.right = self.remove( node.right, node.data )
```

```

    else :
        if node. left != None :
            node = node. left
        else :
            node = node. right
    return node

def search_minimum_value ( self, node):
    if node == None or node. data == 0 :
        return None
    if node. left == None :
        return node
    # Recursively find the minimum from the left subtree
    return self. search_minimum_value( node. left)

def insert ( self, node, new_data):
    if self. __root == None :
        self. __root = Node( new_data, None , None )
        return
    compareValue = new_data - node. data

    #Recursive left subtree, continue to find the insertion position
    if compareValue < 0 :
        if node. left == None :
            node. left = Node( new_data, None , None )
        else :
            self. insert( node. left, new_data)
    elif compareValue > 0 : #Recursive right subtree find the insertion
position
        if node. right == None :
            node. right = Node( new_data, None , None )
        else :
            self. insert( node. right, new_data)

def main ():
    binary_tree = BinaryTree()
    #Constructing a binary search tree
    binary_tree. insert( binary_tree. root, 60 )

```

```

binary_tree.insert( binary_tree.root, 40 )
binary_tree.insert( binary_tree.root, 20 )
binary_tree.insert( binary_tree.root, 10 )
binary_tree.insert( binary_tree.root, 30 )
binary_tree.insert( binary_tree.root, 50 )
binary_tree.insert( binary_tree.root, 80 )
binary_tree.insert( binary_tree.root, 70 )
binary_tree.insert( binary_tree.root, 90 )

print ( "\ndelete node is: 10" )
binary_tree.remove( binary_tree.root, 10 )
print ( "\nIn-order traversal binary tree" )
binary_tree.inOrder( binary_tree.root)

print ( "\n-----" )

print ( "\ndelete node is: 20" )
binary_tree.remove( binary_tree.root, 20 )
print ( "\nIn-order traversal binary tree" )
binary_tree.inOrder( binary_tree.root)

print ( "\n-----" )

print ( "\ndelete node is: 40" )
binary_tree.remove( binary_tree.root, 40 )
print ( "\nIn-order traversal binary tree" )
binary_tree.inOrder( binary_tree.root)

if __name__ == "__main__" :
    main()

```

Result:

delete node is: 10

In-order traversal binary tree
 20 , 30 , 40 , 50 , 60 , 70 , 80 , 90 ,

delete node is: 20

In-order traversal binary tree
30 , 40 , 50 , 60 , 70 , 80 , 90 ,

delete node is: 40

In-order traversal binary tree
30 , 50 , 60 , 70 , 80 , 90 ,

Binary Heap Sorting

Binary Heap Sorting:

The value of the non-terminal node in the binary tree is not greater than the value of its left and right child nodes.

Small top heap : $k_i \leq k_{2i}$ and $k_i \leq k_{2i+1}$

Big top heap : $k_i \geq k_{2i}$ and $k_i \geq k_{2i+1}$

Parent node subscript = $(i-1)/2$

Left subnode subscript = $2*i+1$

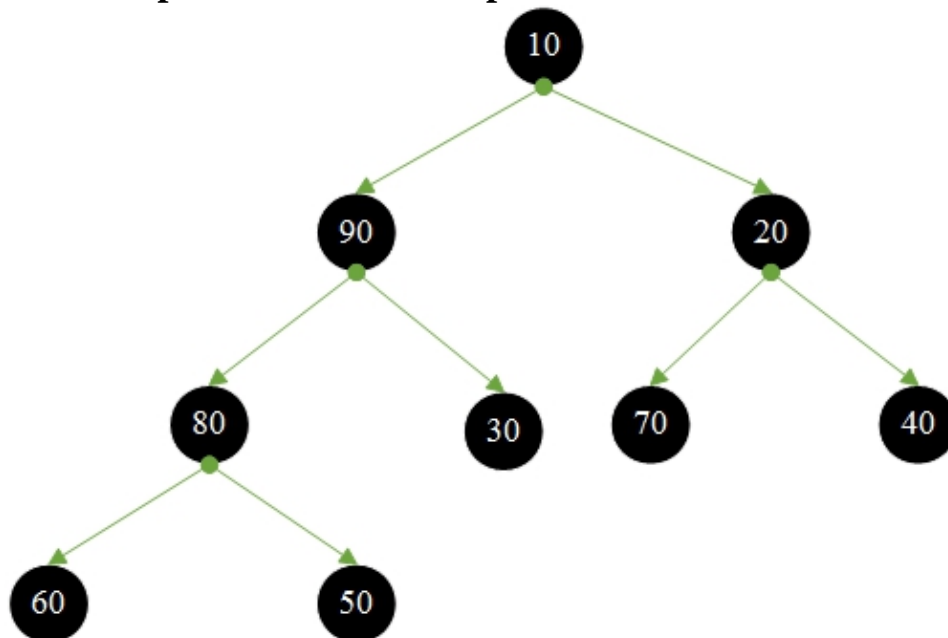
Right subnode subscript = $2*i+2$

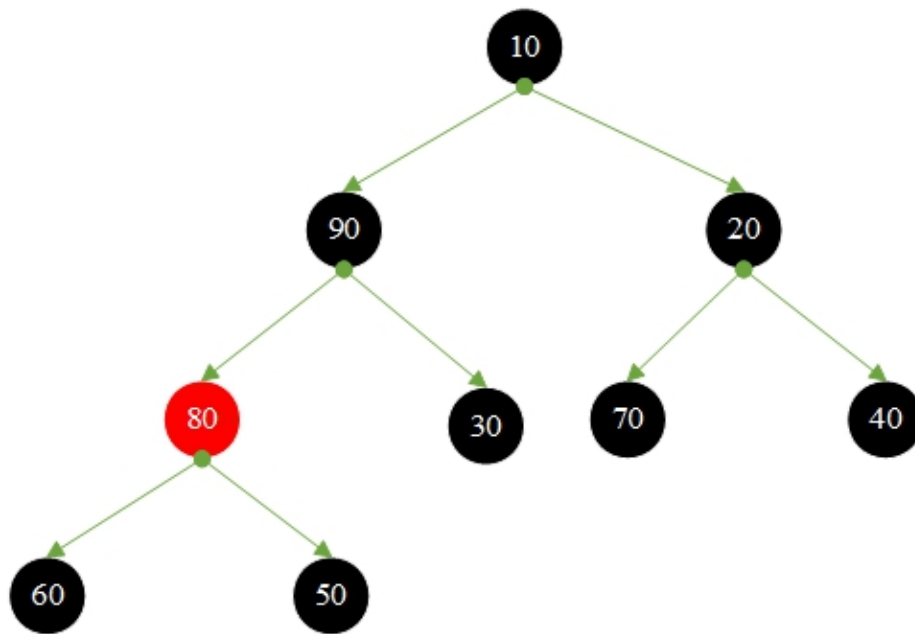
Heap sorting process:

1. Build a heap
2. After outputting the top element of the heap, adjust from top to bottom, compare the top element with the root node of its left and right subtrees, and swap the smallest element to the top of the heap; then adjust continuously until the leaf nodes to get new heap.

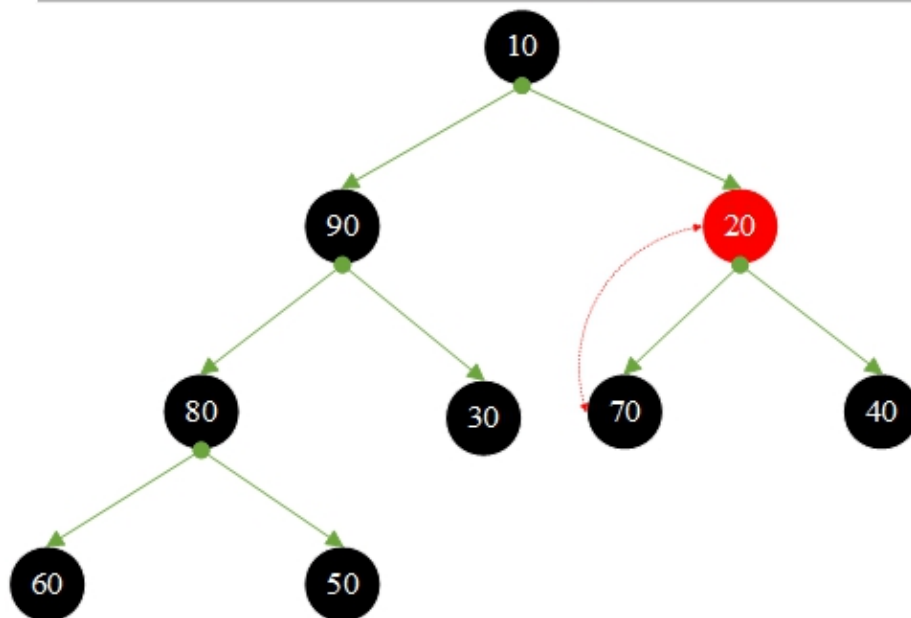
1. **{10, 90, 20, 80, 30, 70, 40, 60, 50}** build heap and then heap sort output.

Initialize the heap and build the heap

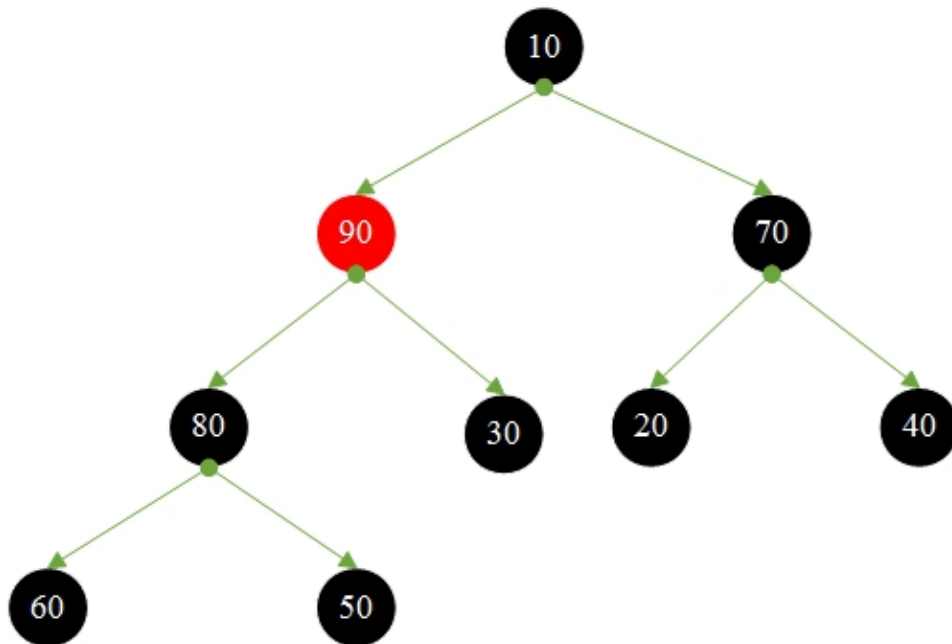




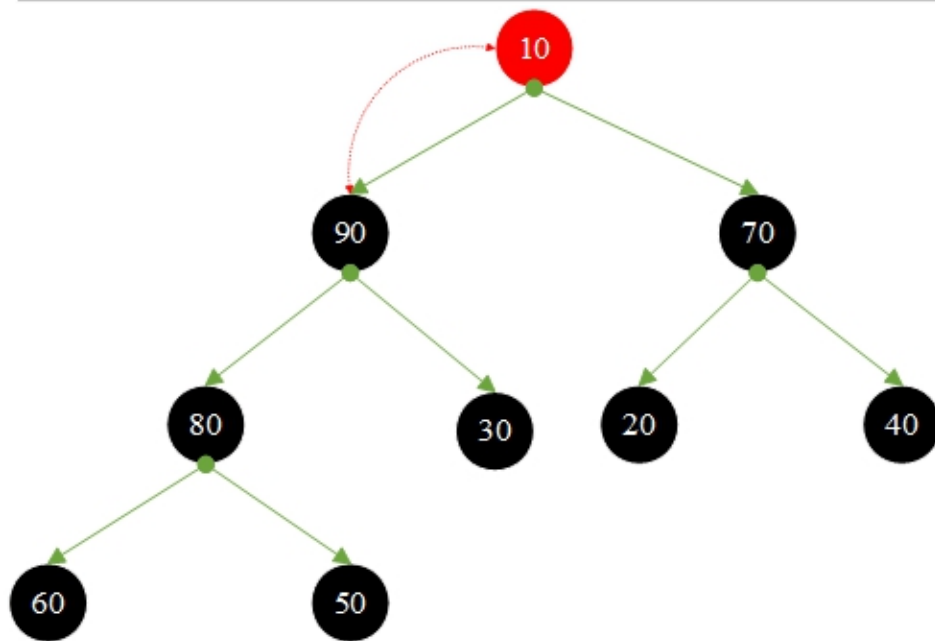
Not Leaf Node = 80 > left = 60 , 80 > right = 50 No need to move



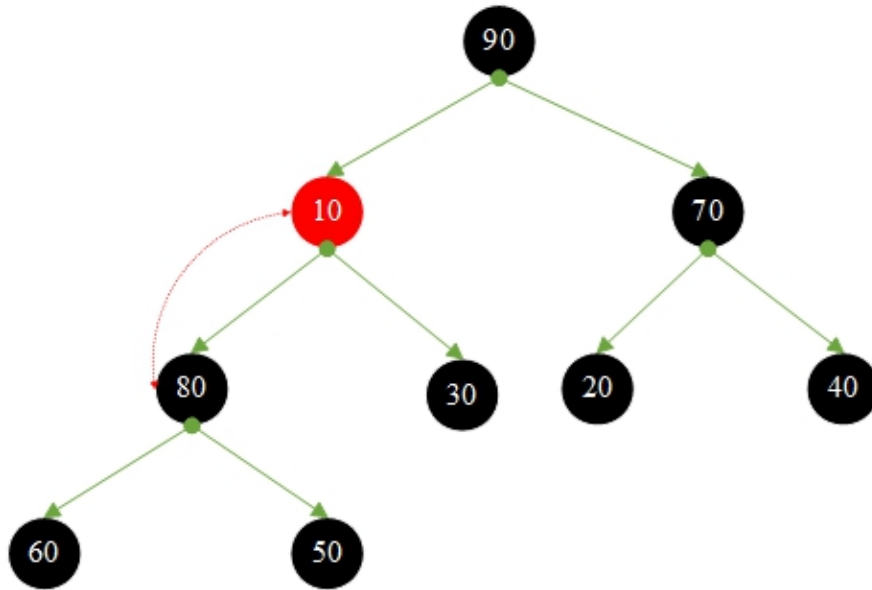
Not Leaf Node = 20 < left = 70 , 70 > right = 40 , 20 swap with 70



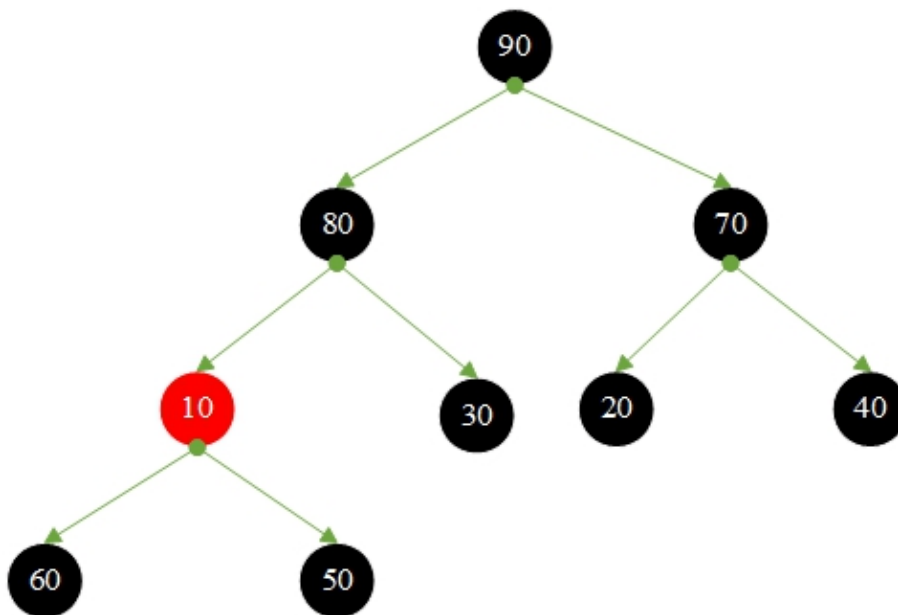
Not Leaf Node = 90 > left = 80 , 80 > right = 30 No need to move



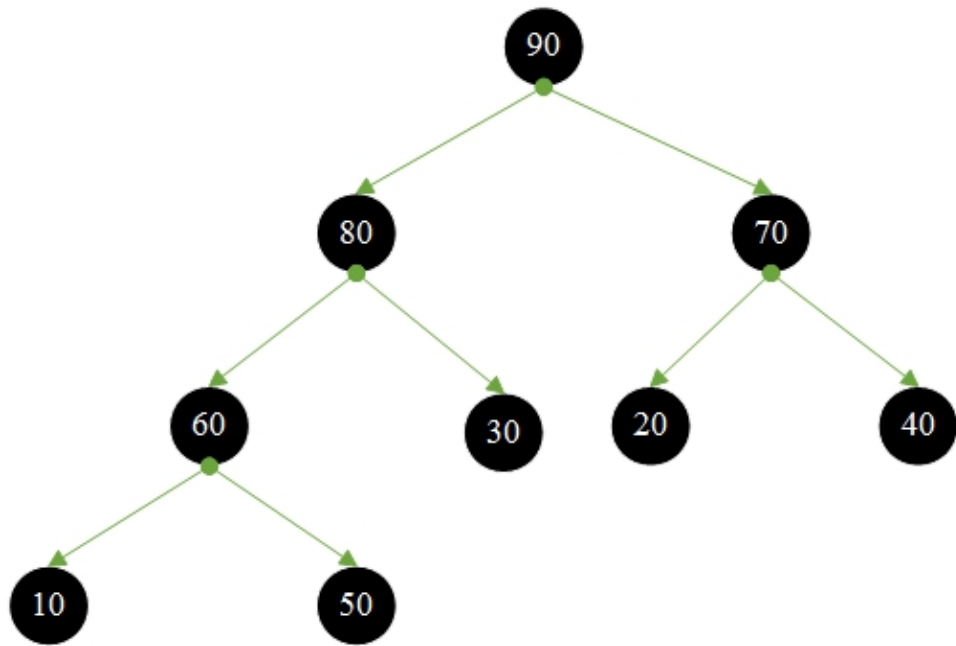
Not Leaf Node = 10 < left = 90 , 90 > right = 70 , 10 swap with 90



Still Not Leaf Node = 10 < left = 80 , 80 > right = 30 , 10 swap with 80

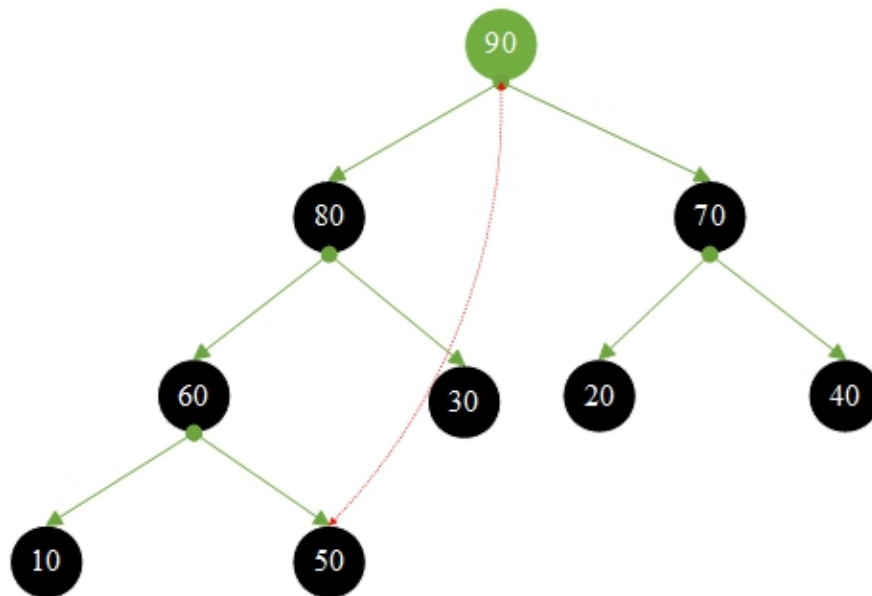


Still Not LeafNode = 10 < left = 60 , 60 > right = 50 , 10 swap with 60

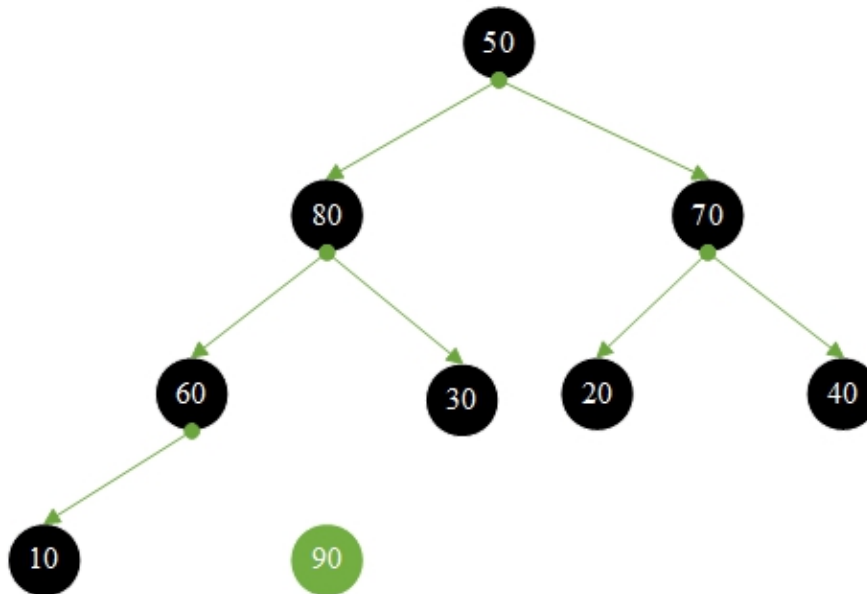


Create the heap finished

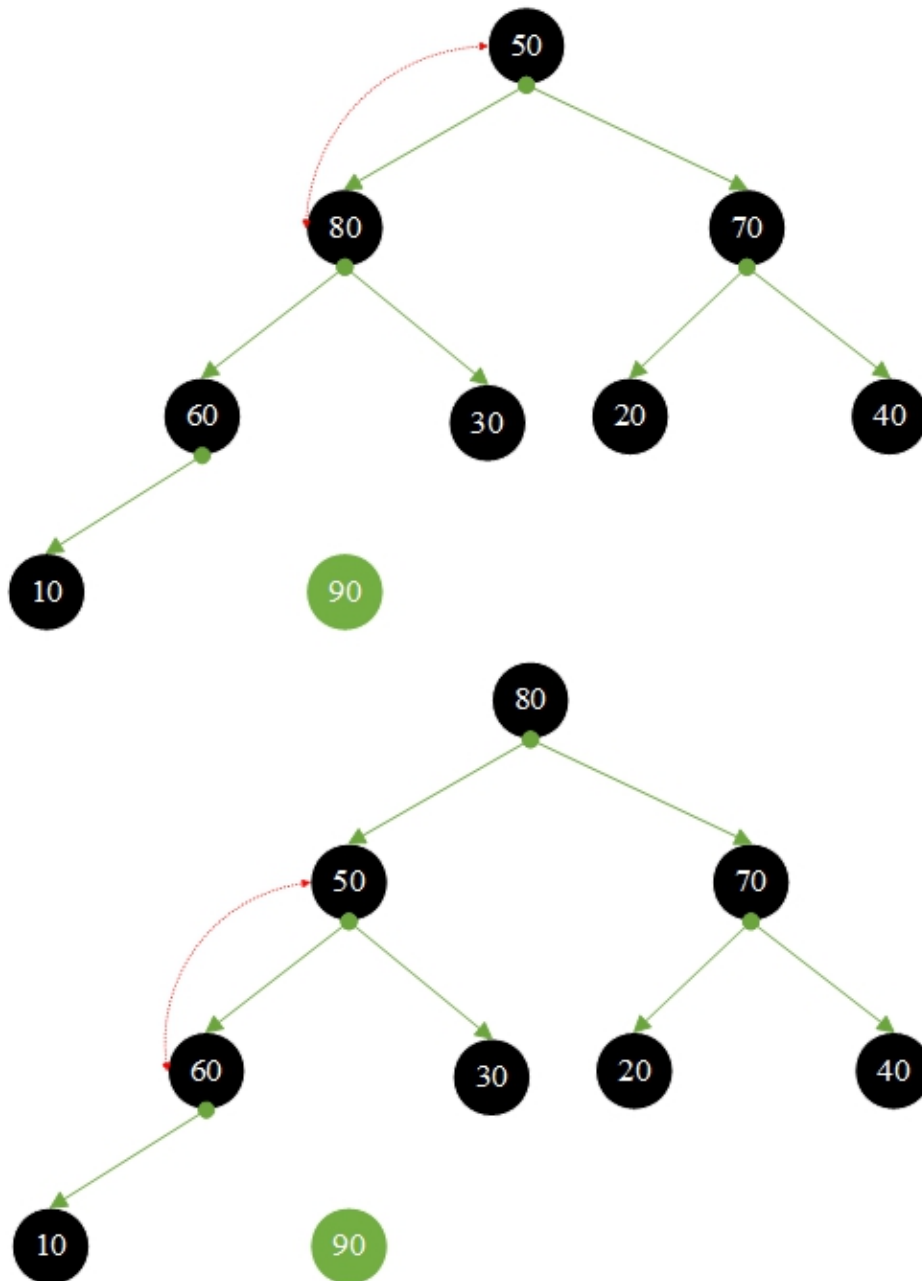
2. Start heap sorting

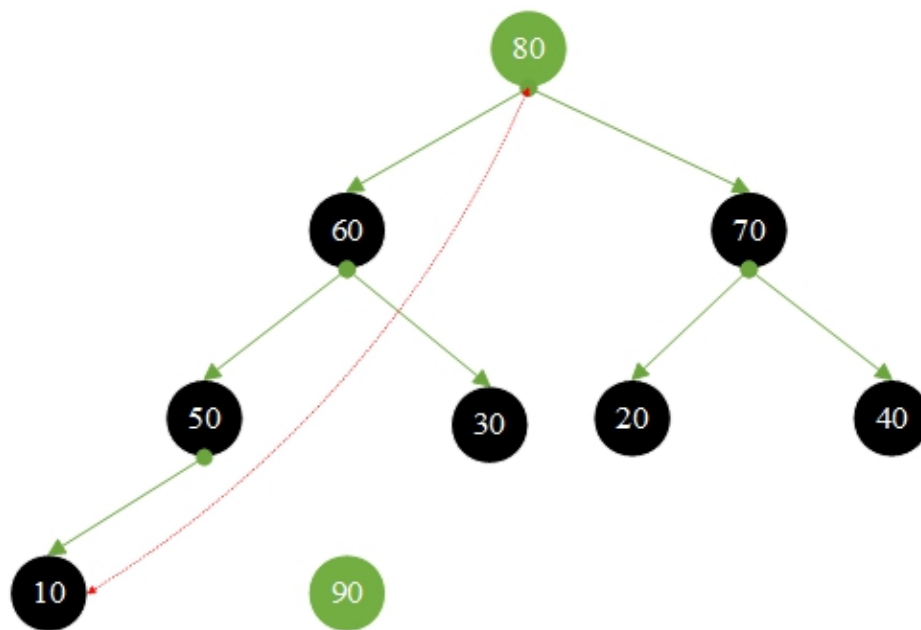


root = 90 and tail = 50 are exchanged

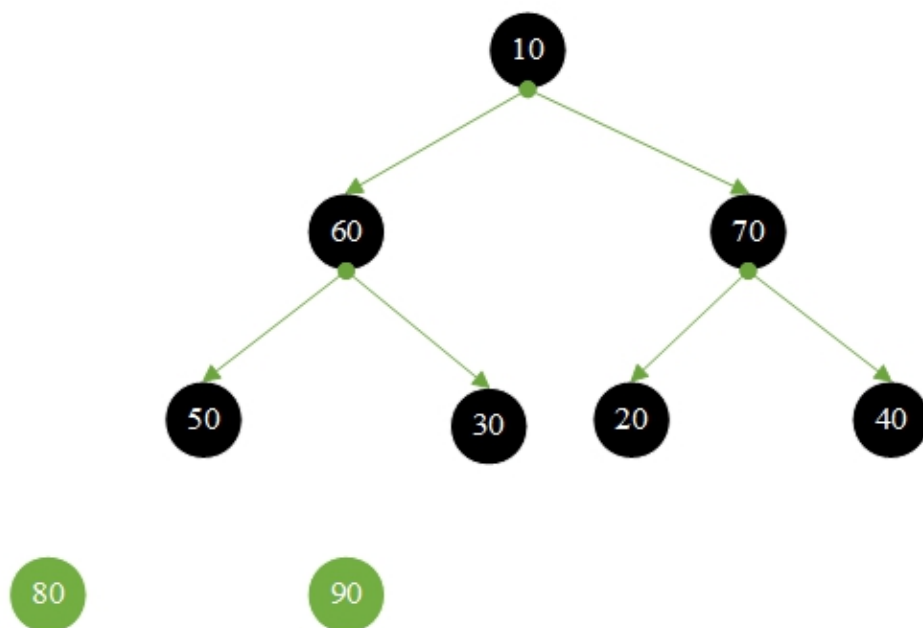


adjust the heap

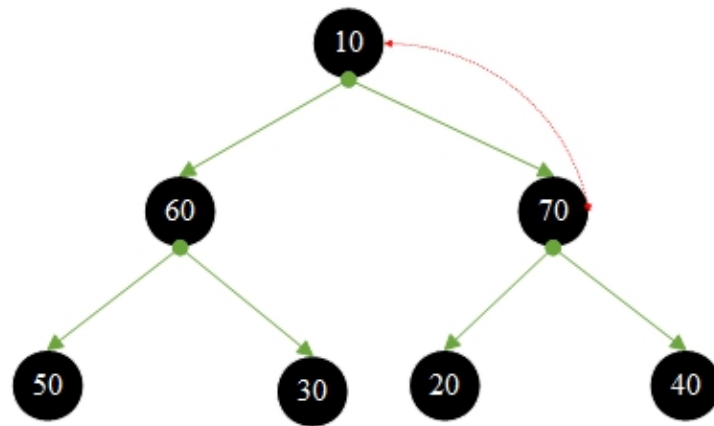




root = 80 and tail = 10 are exchanged

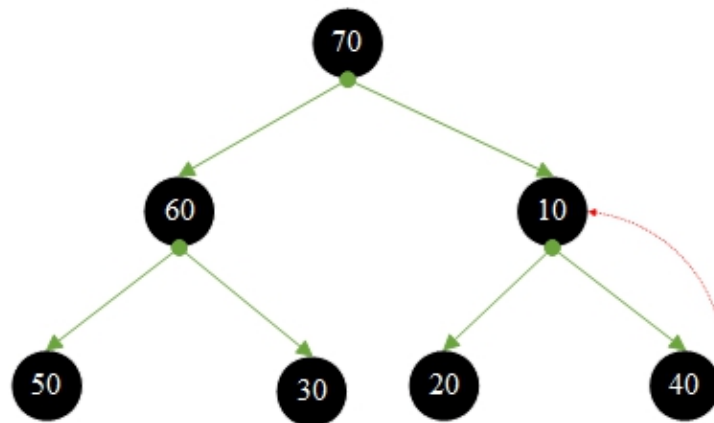


adjust the heap



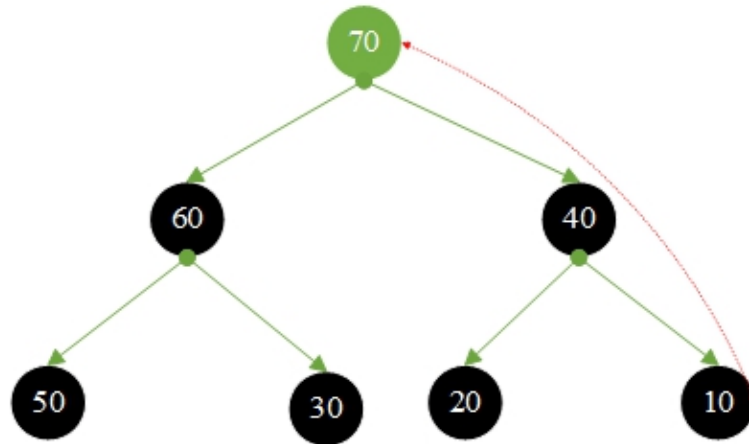
80

90



80

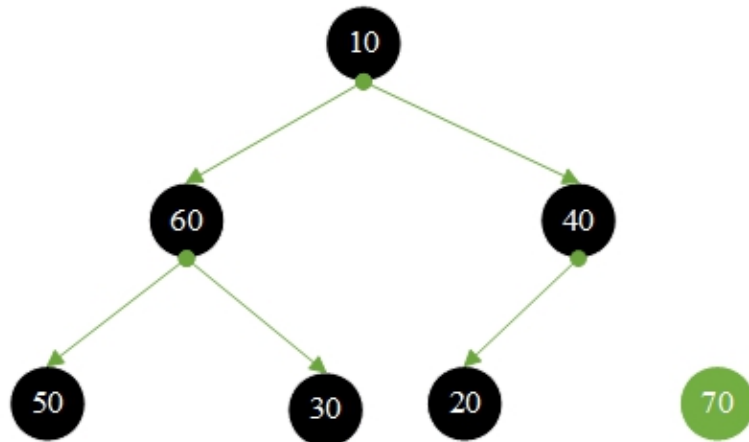
90



80

90

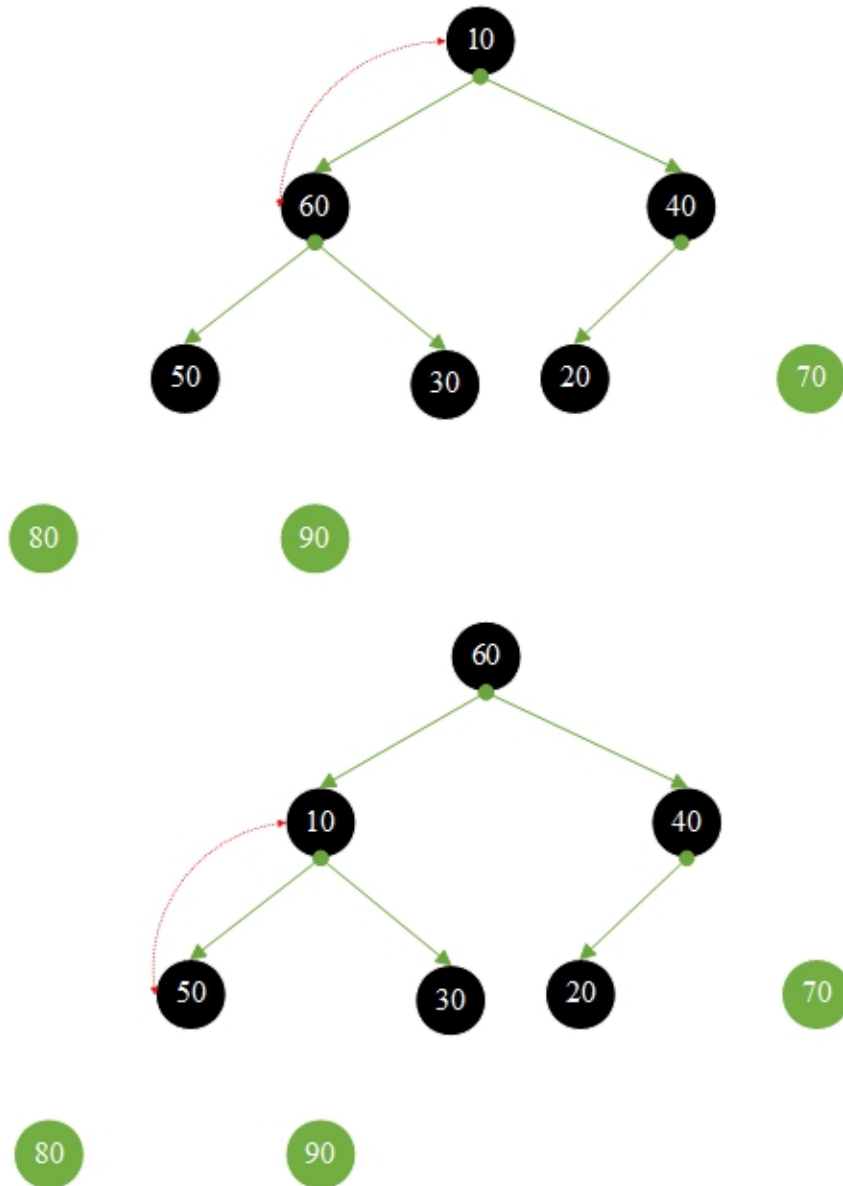
root = 70 and tail = 10 are exchanged

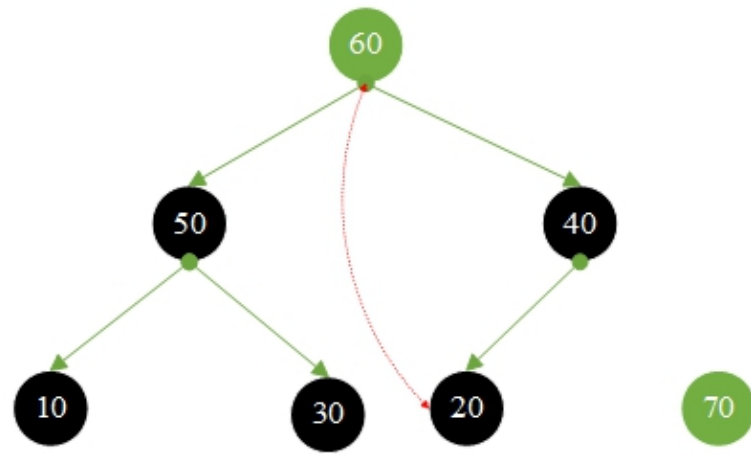


80

90

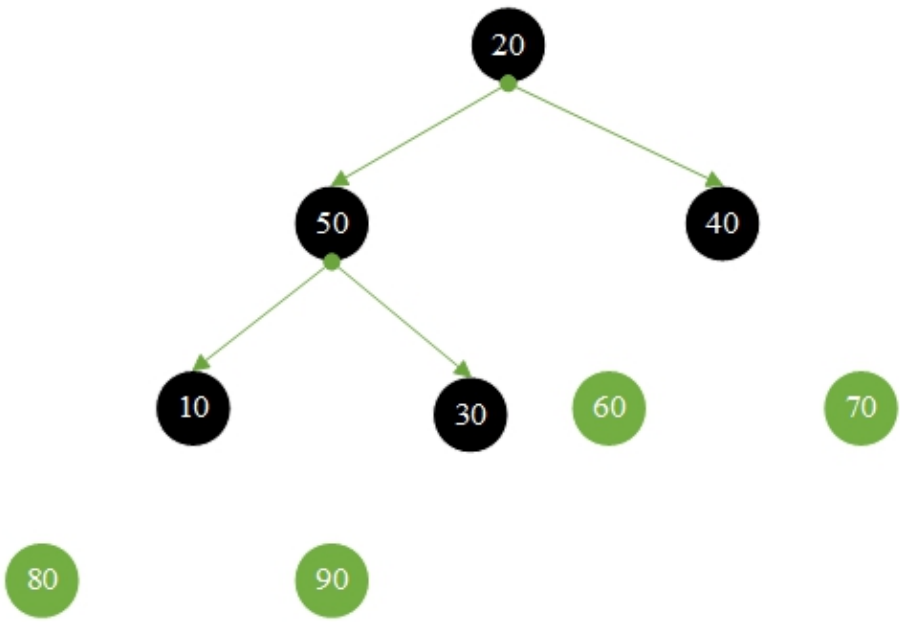
adjust the heap



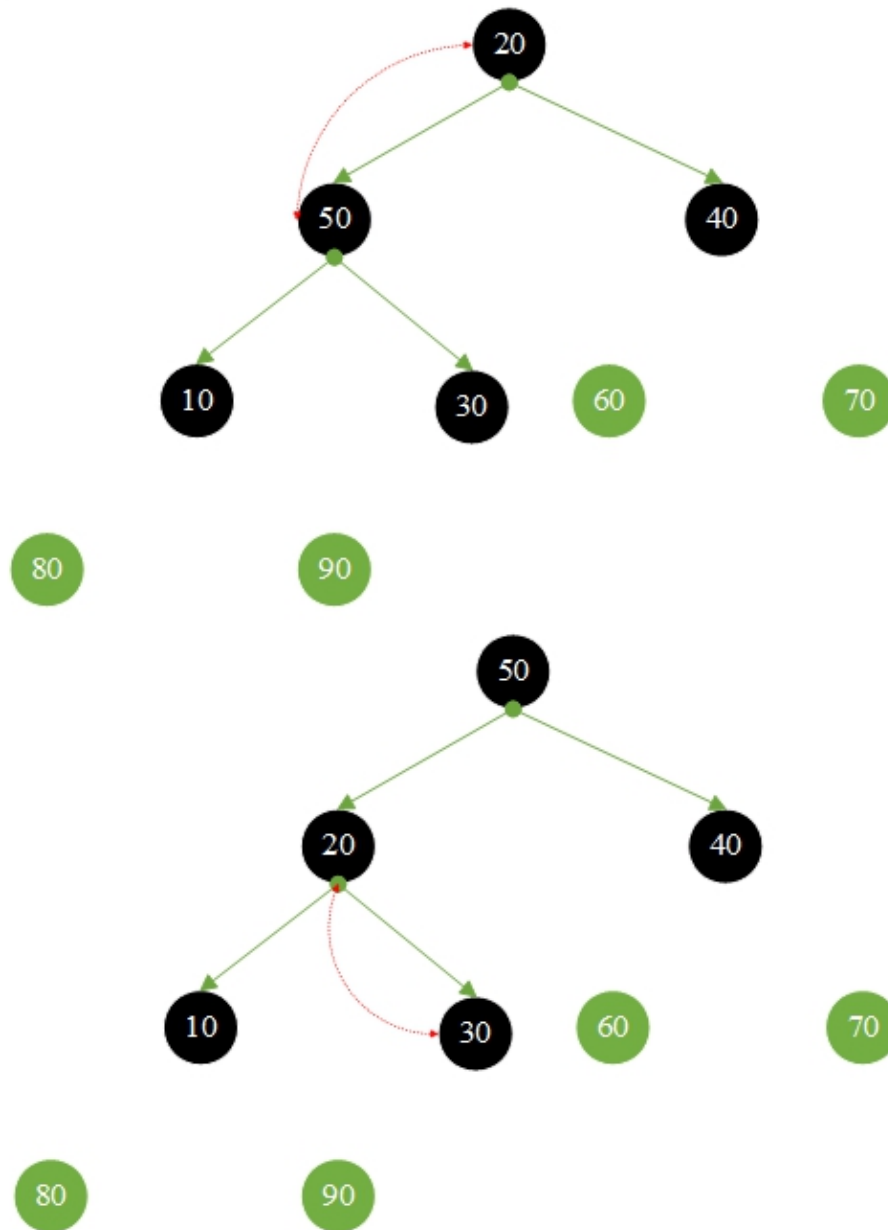


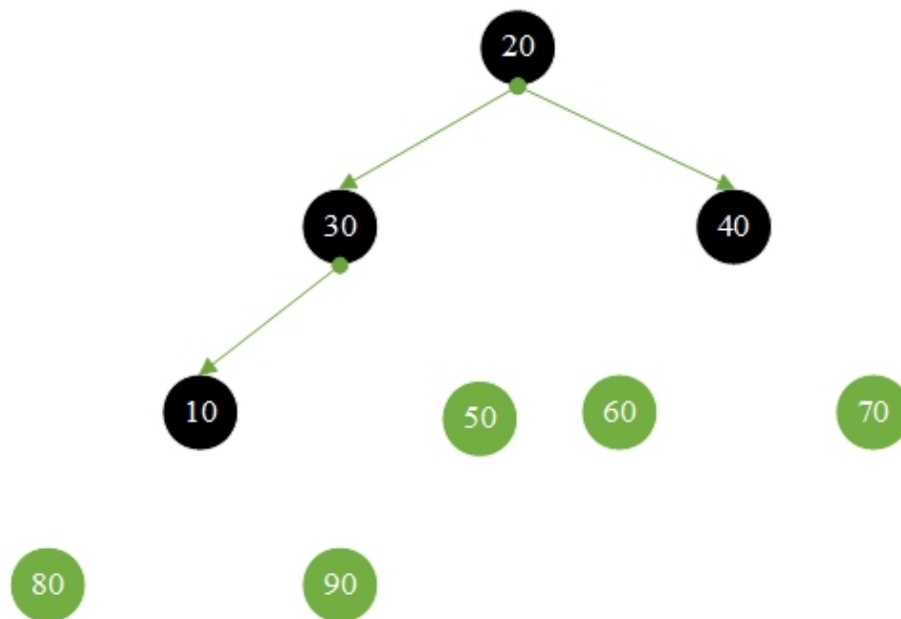
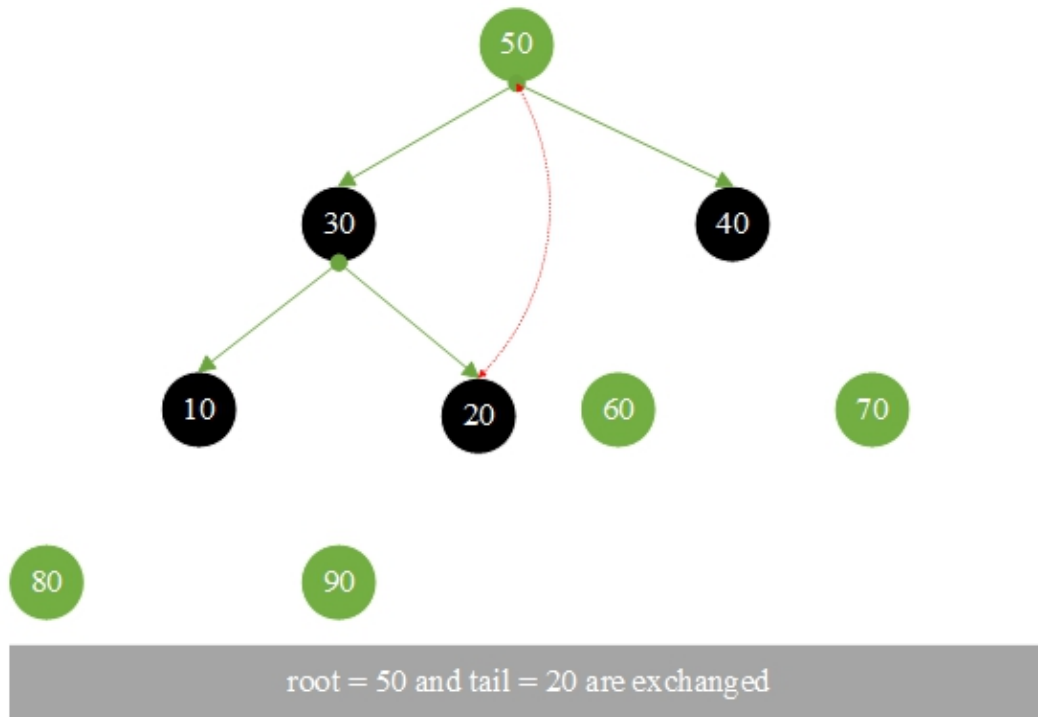
80 90

root = 60 and tail = 20 are exchanged

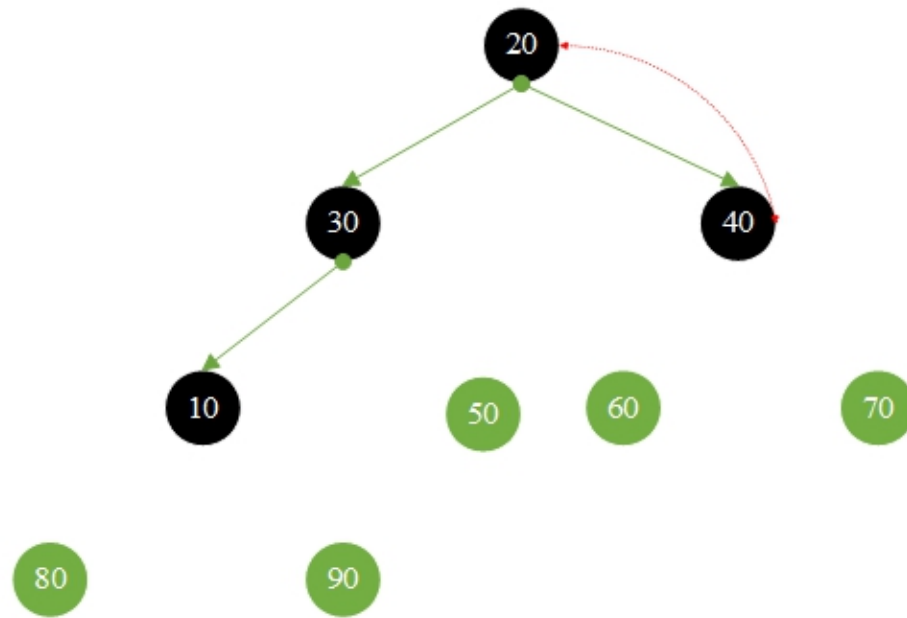


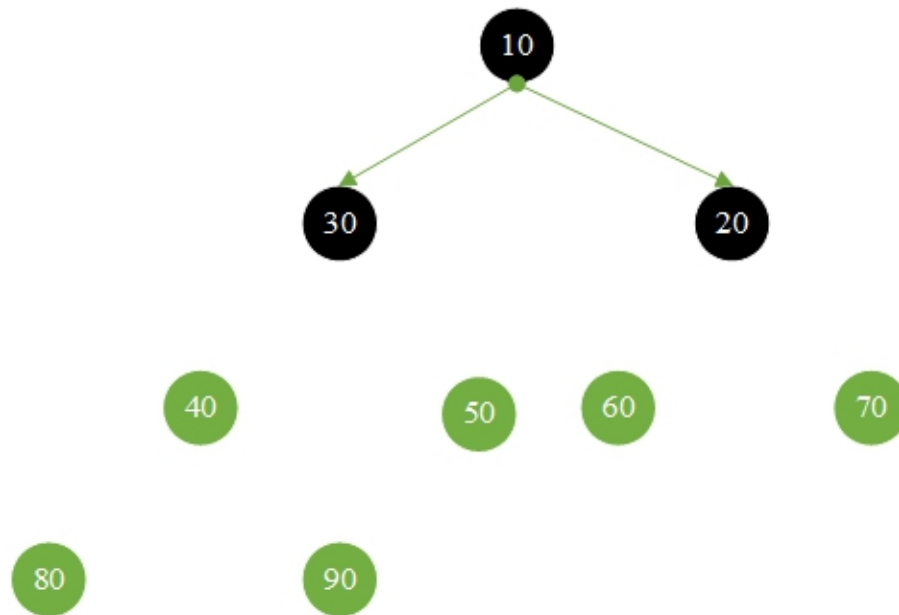
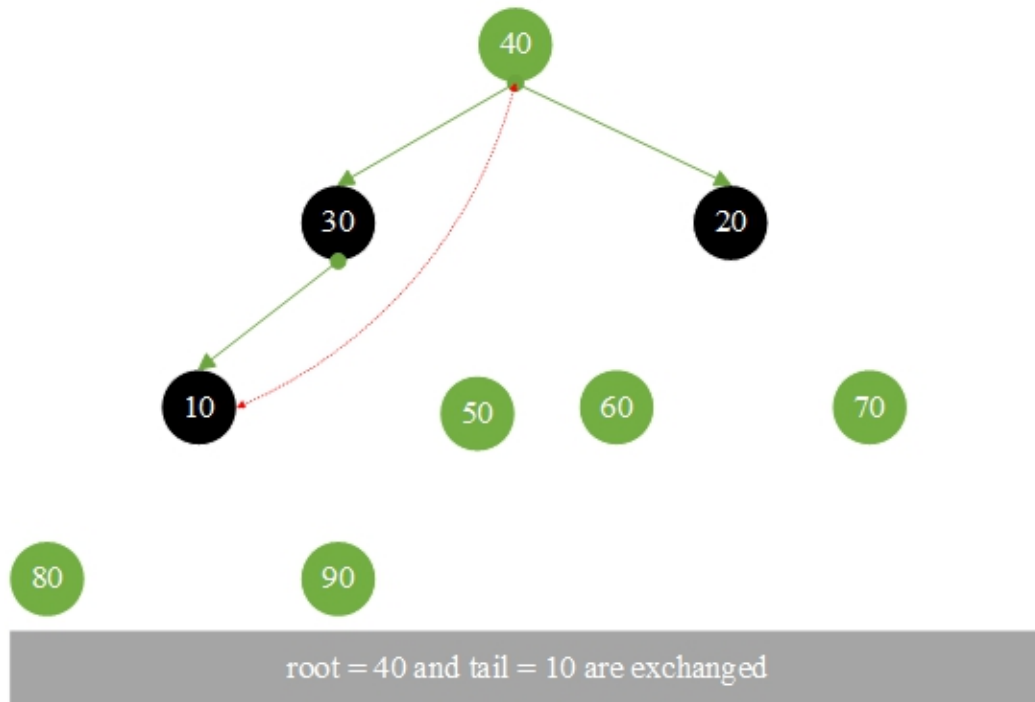
adjust the heap



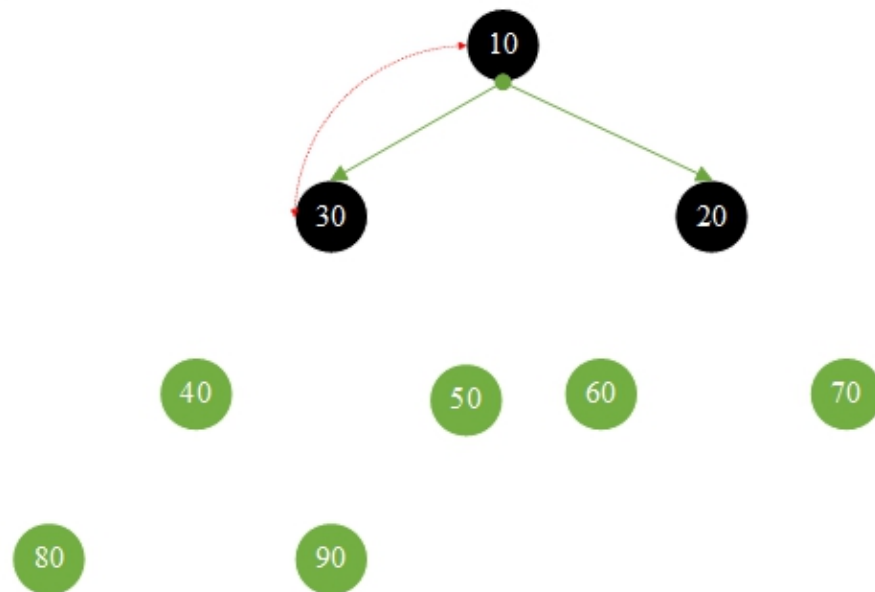


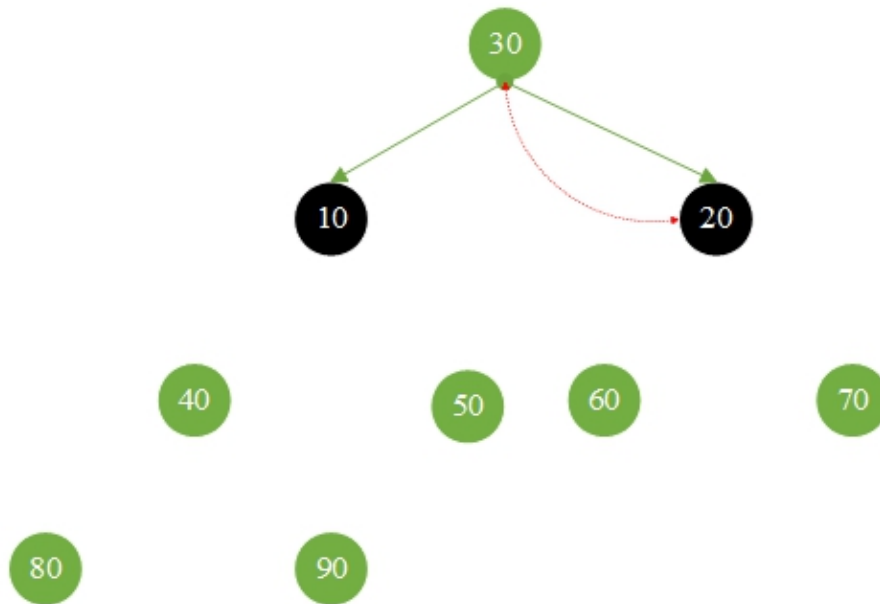
adjust the heap



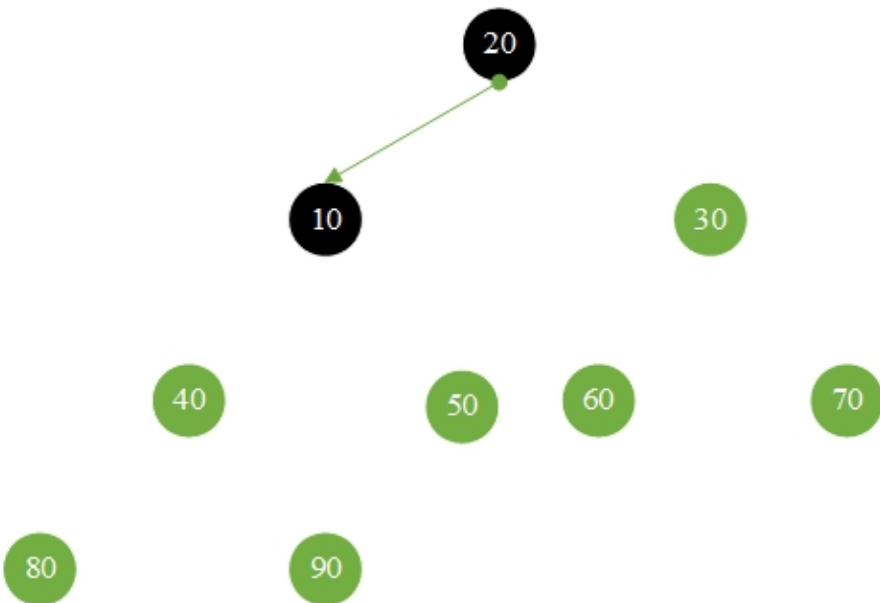


adjust the heap

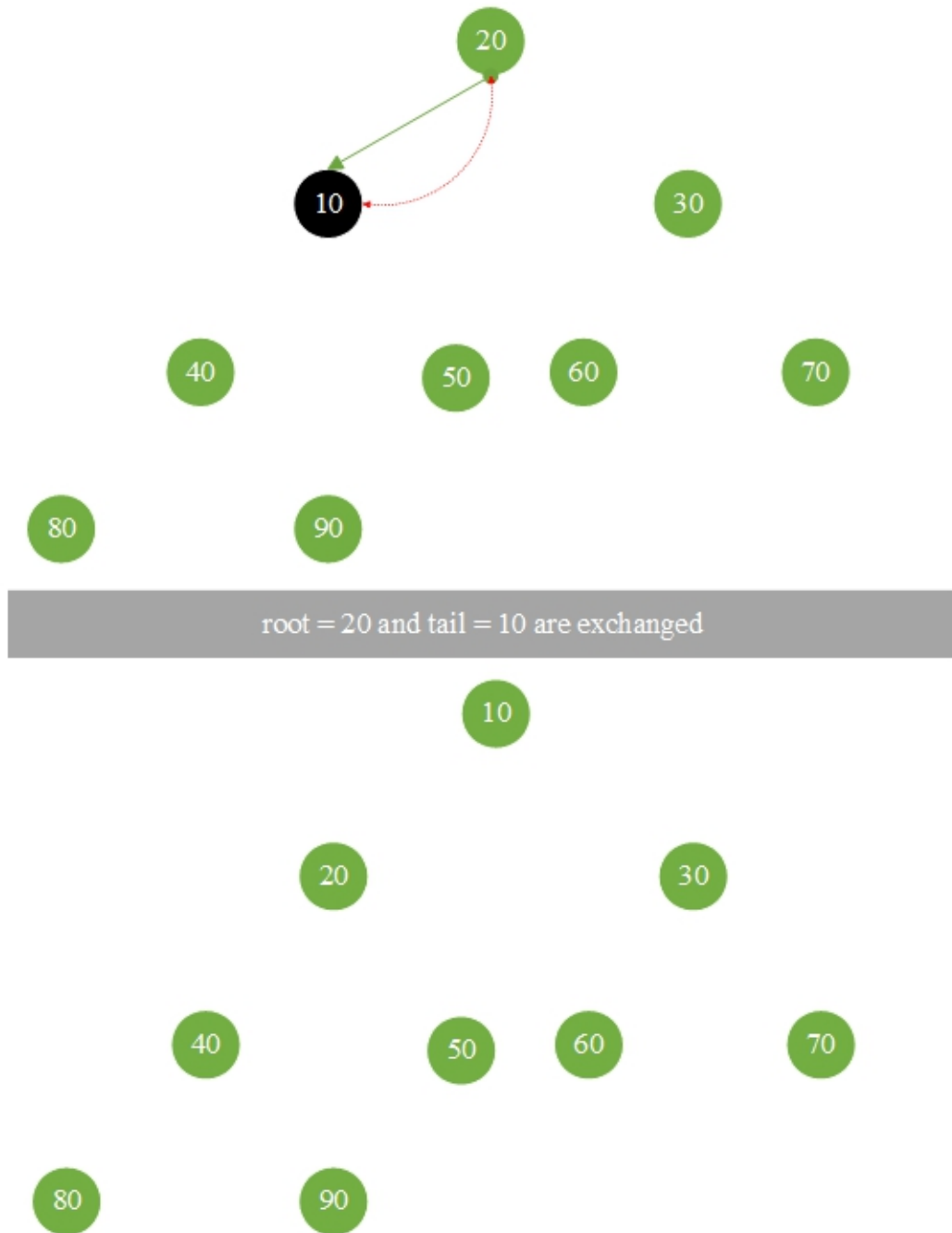




root = 30 and tail = 20 are exchanged



no need adjust the heap



Heap sort result



test_heap_sort.py

```
class HeapSort :
```

```
    __array = []
```

```
    #Initialize the heap
```

```
    def create_heap ( self, array):
```

```
        self.__array = array
```

```
        # Build a heap, (array.length - 1) / 2 scan half of the nodes with  
child nodes
```

```
        length = len( array)
```

```
        for i in range(( length - 1 ) // 2 , - 1 , - 1 ):
```

```
            self.adjust_heap( i, length - 1 )
```

```
    # Adjustment heap
```

```
    def adjust_heap ( self, current_index, max_length):
```

```
        no_leaf_value = self.__array[ current_index] # Current non-leaf  
node
```

```
        # 2 * current_index + 1 Current left subtree subscript
```

```
        j = 2 * current_index + 1
```

```
        while ( j <= max_length):
```

```
            if ( j < max_length and self.__array[ j] < self.__array[ j + 1 ]):
```

```
                j += 1 # j Large subscript
```

```
            if ( no_leaf_value >= self.__array[ j]):
```

```
                break
```

```
            self.__array[ current_index] = self.__array[ j] # Move up to  
the parent node
```

```
            current_index = j
```



```

        j = current_index * 2 + 1

        self.__array[ current_index ] = no_leaf_value # To put in the
position

def heap_sort ( self):
    length = len( self.__array)
    for i in range( length - 1 , 0 , - 1 ):
        temp = self.__array[ 0 ]
        self.__array[ 0 ] = self.__array[ i ]
        self.__array[ i ] = temp
        self.adjust_heap( 0 , i - 1 )

def main ():
    heap_sort = HeapSort()
    scores = [ 10 , 90 , 20 , 80 , 30 , 70 , 40 , 60 , 50 ]
    print ( "Before building a heap : " )
    for score in scores:
        print ( score, " , " , end= "" )
    print ( "\n\n" )

    print ( "After building a heap : " )
    heap_sort.create_heap( scores)
    for score in scores:
        print ( score, " , " , end= "" )
    print ( "\n\n" )

    print ( "After heap sorting : " )
    heap_sort.heap_sort();
    for score in scores:
        print ( score, " , " , end= "" )

if __name__ == "__main__" :
    main ()

```

Result:

Before building a heap :

10 , 90 , 20 , 80 , 30 , 70 , 40 , 60 , 50 ,

After building a heap :

90 , 80 , 70 , 60 , 30 , 20 , 40 , 10 , 50 ,

After heap sorting :

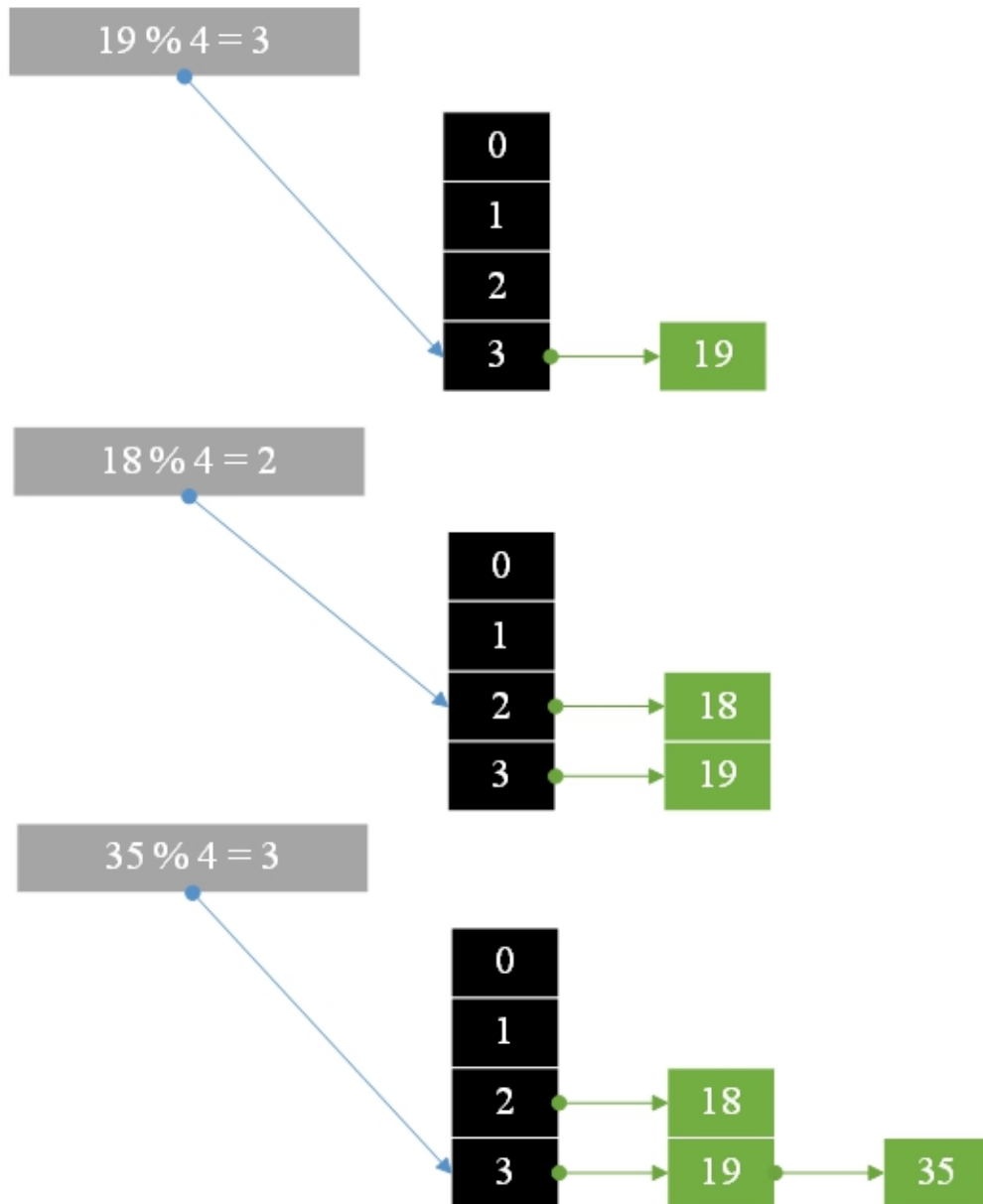
10 , 20 , 30 , 40 , 50 , 60 , 70 , 80 , 90 ,

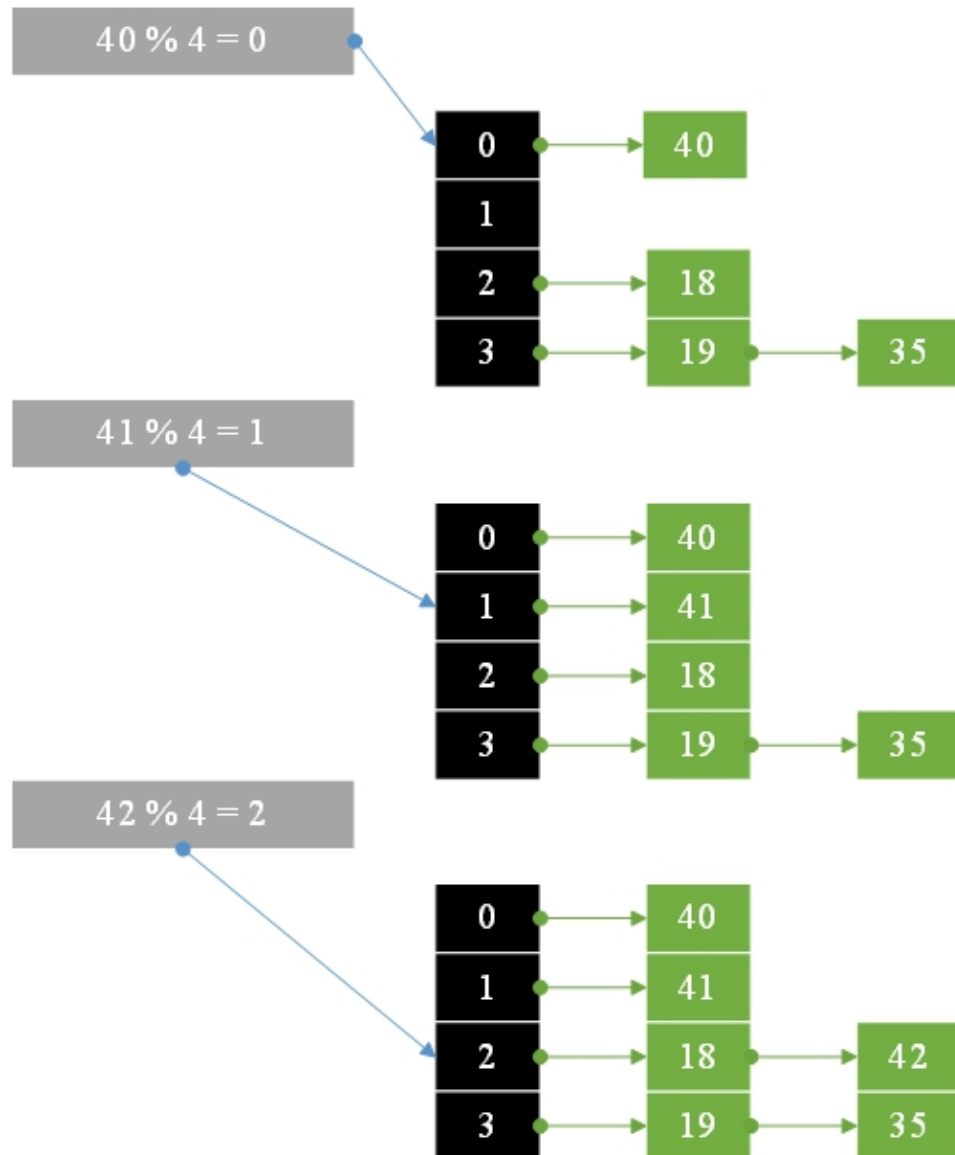
Hash Table

Hash Table:

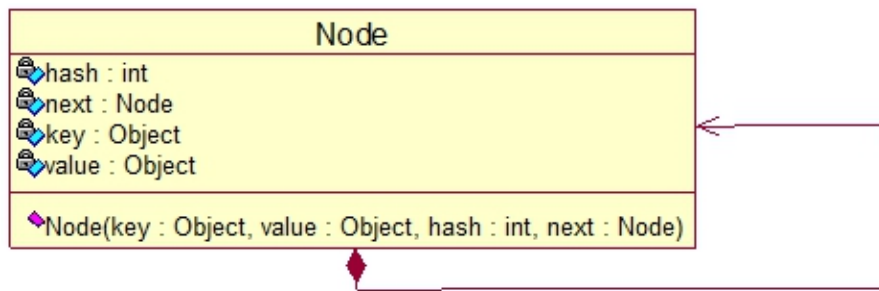
Access by mapping key => values in the table.

1. Map {19, 18, 35, 40, 41, 42} to the HashTable mapping rule **key % 4**





2. Implement a Hashtable



```
class Node :
```

```
    key = None
    value = None
    hash = 0
    next = None
```

```
    def __init__ ( self, key, value, hash, next):
        self. key = key
        self. value = value
        self. hash = hash
        self. next = next
```

hashtable.py

```
import math

class Node :
    key = None
    value = None
    hash = 0
    next = None

    def __init__ ( self, key, value, hash, next):
        self.key = key
        self.value = value
        self.hash = hash
        self.next = next

class Hashtable :

    __table = []
    __size = 0

    # Initialize the Hashtable size
    def __init__ ( self, capacity):
        self.__table = [[ 0 for i in range( capacity)]]
        self.__size = 0

    @property
    def size ( self):
        return self.__size

    def is_empty ( self):
        if self.__size == 0 :
            return True
        else :
            return False

    #Calculate the hash value according to the key hash algorithm
    def hash_code ( self, key ):
```

```
    avg = hash( key ) * ( math. pow( 5 , 0.5 ) - 1 ) / 2 #hash policy for  
middle-square method
```

```
    numeric = avg - math. floor( avg )  
    return int( math. floor( numeric * len( self. __table )))
```

```
def put ( self, key, value):
```

```
    if key == None :  
        return
```

```
    hash = self. hash_code( key )  
    new_node = Node( key, value, hash, None )
```

```
    node = self. __table[ hash ]  
    while node != None and hash != 0 :
```

```
        if node. key == key:  
            node. value = value  
            return
```

```
        node = node. next
```

```
    new_node. next = self. __table[ hash ]  
    self. __table[ hash ] = new_node  
    self. __size += 1
```

```
def get ( self, key):
```

```
    if key == None :  
        return None
```

```
    hash = self. hash_code( key )  
    node = self. __table[ hash ]
```

```
    while node != None : #Get value according to key
```

```
        if node. key == key:  
            return node. value
```

```
        node = node. next
```

```
    return None
```

```
def main ():
```

```
    table = Hashtable( 16 )
```

```
    table. put( "david" , "Good Boy Keep Going" )
```

```
    table. put( "grace" , "Cute Girl Keep Going" )
```

```
print ( "david => " , table.get( "david" ))  
print ( "grace => " , table.get( "grace" ))  
  
if __name__ == "__main__" :  
    main()
```

Result:

david => Good Boy Keep Going

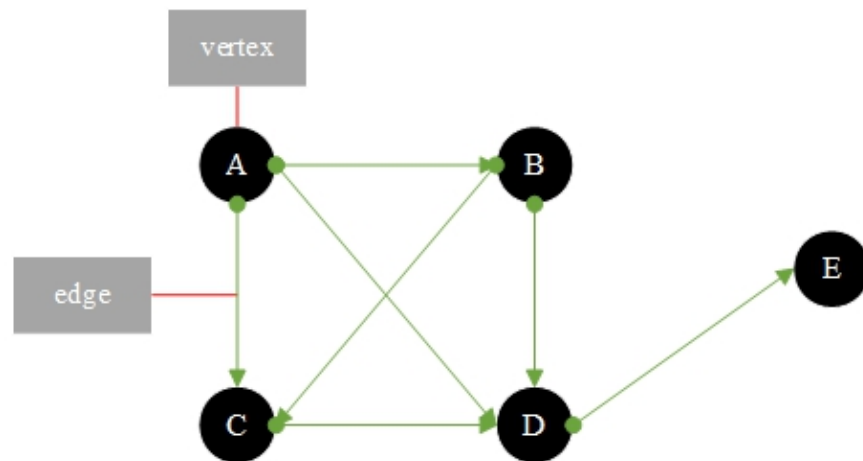
grace => Cute Girl Keep Going

Directed Graph and Depth-First Search

Directed Graph:

The data structure is represented by an adjacency matrix (that is, a two-dimensional array) and an adjacency list. Each node is called a vertex, and two adjacent nodes are called edges.

Directed Graph has direction : $A \rightarrow B$ and $B \rightarrow A$ are different



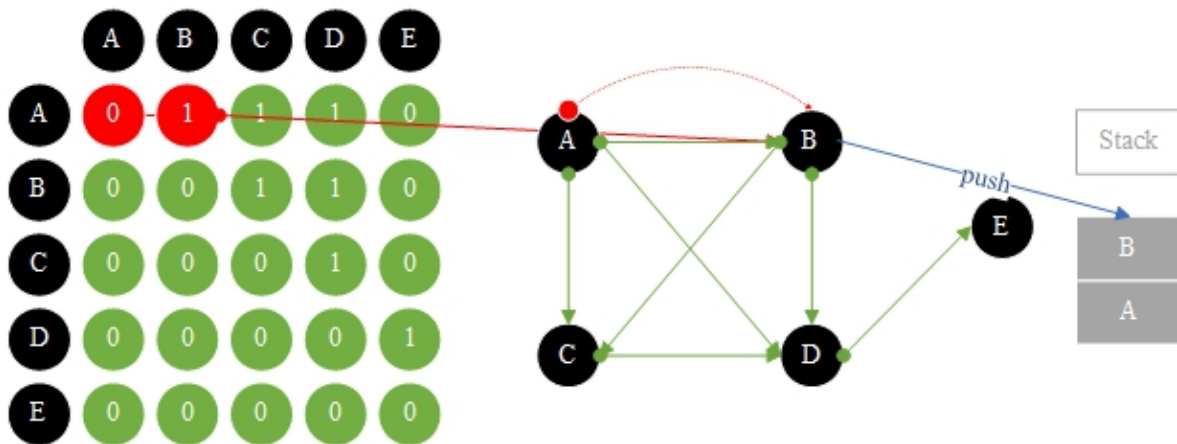
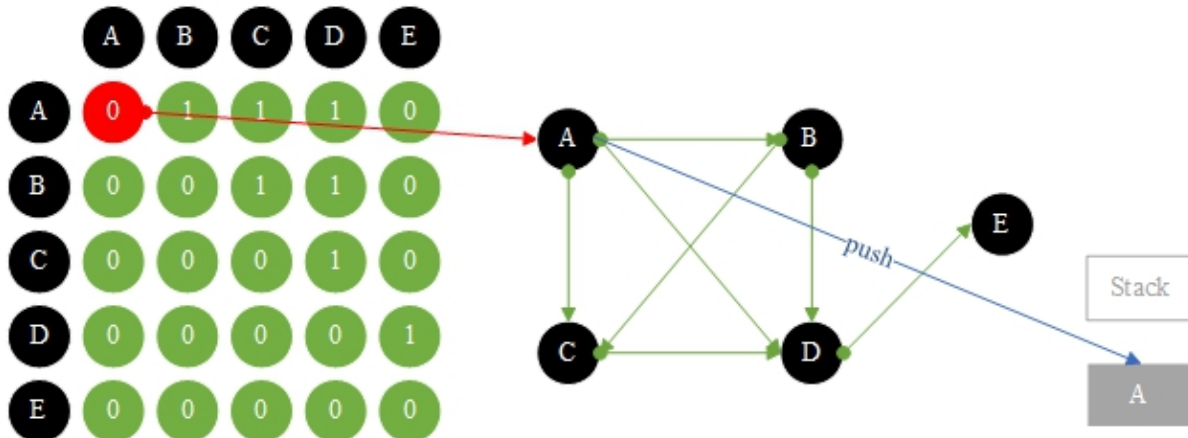
1. The adjacency matrix is described above:

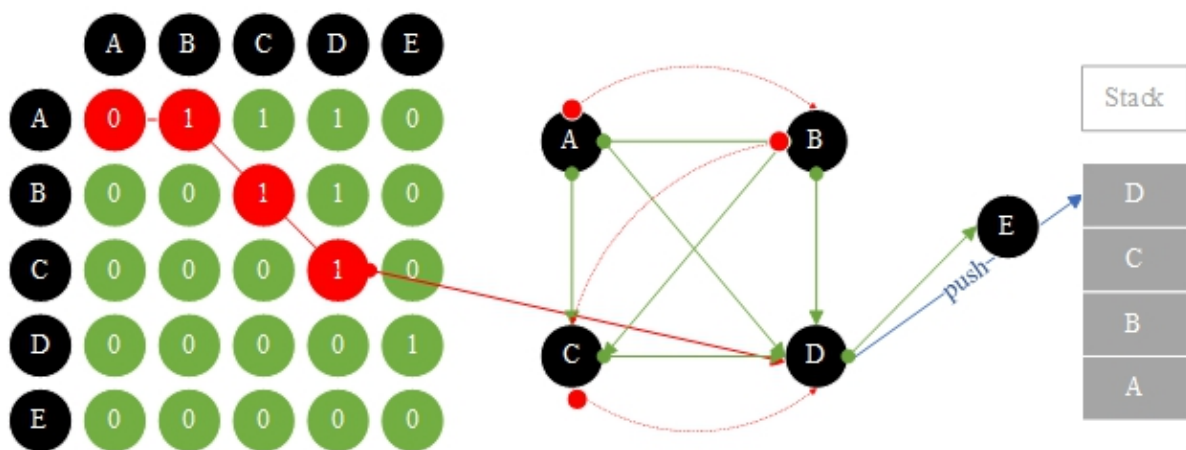
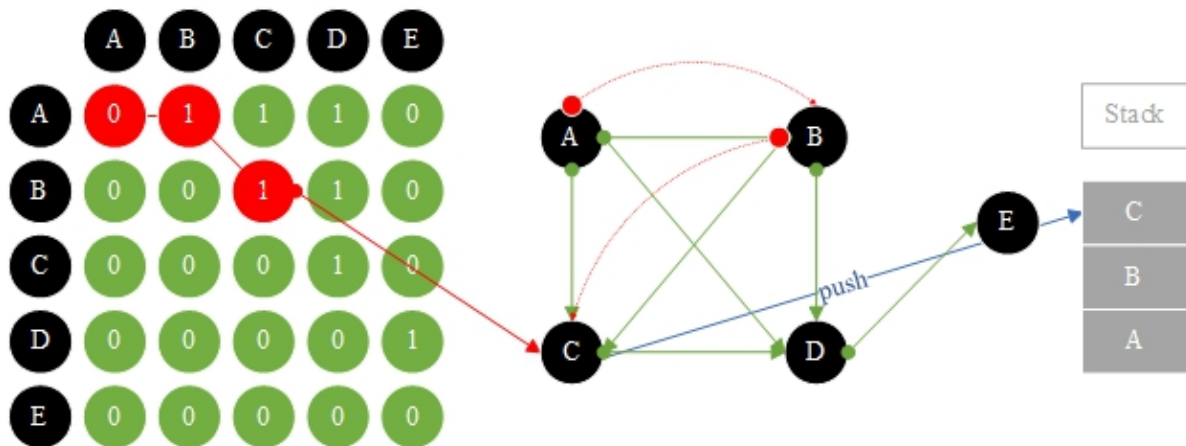
The total number of vertices is a two-dimensional array size, if have value of the edge is **1** , otherwise no value of the edge is **0** .

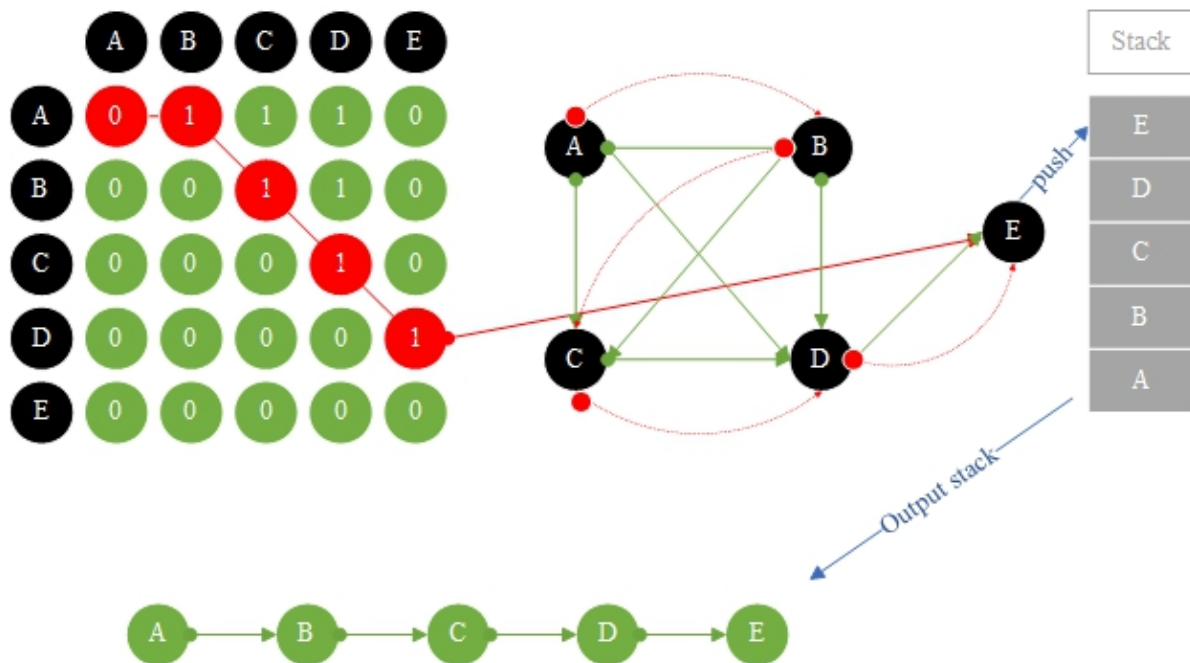
	A	B	C	D	E
A	0	1	1	1	0
B	0	0	1	1	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	0	0	0

2. Depth-First Search:

Look for the neighboring edge node B from A and then find the neighboring node C from B and so on until all nodes are found **A -> B -> C**
-> D -> E.







node.py

```
class Node :  
    data = ''  
    prev = None  
    next = None  
  
    def __init__ ( self, data, prev, next):  
        self.data = data  
        self.prev = prev  
        self.next = next
```

vertex.py

```
class Vertex :  
    data = ''  
    visited = False # Have you visited  
  
    def __init__ ( self, data, visited):  
        self.data = data  
        self.visited = visited
```

stack.py

```
from node import Node

class Stack :
    __head = None
    __count = 0

    def push ( self, element):
        if self.__head == None :
            self.__head = Node( element, None ,
None )
        else :
            new_node = Node( element, None ,
None )
            new_node.next = self.__head
            self.__head.prev = new_node
            self.__head = new_node
            self.__count += 1

    def pop ( self):
        p = self.__head
        self.__head = self.__head.next

        # Delete node
        p.next = None
        p.prev = None

        self.__count -= 1
        return p.data

    def peek ( self):
        return self.__head.data

    def size ( self):
        return self.__count

    def is_empty ( self):
        if self.__count == 0 :
```

```
return True  
else :  
return False
```

graph.py

test_graph.py

```
from graph import Graph

def print_graph ( graph):
    print ( "Two-dimensional array traversal output vertex edge and
adjacent array: \n " )
    length = len( graph. get_vertexs())
    print ( "    ", end= "" )
    for i in range( 0 , length):
        print ( graph. get_vertexs()[ i]. data , " " , end= "" )
    print ( "\n" )

    length = len( graph. get_adjacency_matrix())
    for i in range( 0 , length):
        print ( graph. get_vertexs()[ i]. data , " " , end= "" )
        for j in range( 0 , length):
            print ( graph. get_adjacency_matrix()[ i][ j] , " " , end= "" )
        print ( "\n" )

def main ():
    graph = Graph( 5 )

    # Add Vertex
    graph. add_vertex( "A" )
    graph. add_vertex( "B" )
    graph. add_vertex( "C" )
    graph. add_vertex( "D" )
    graph. add_vertex( "E" )

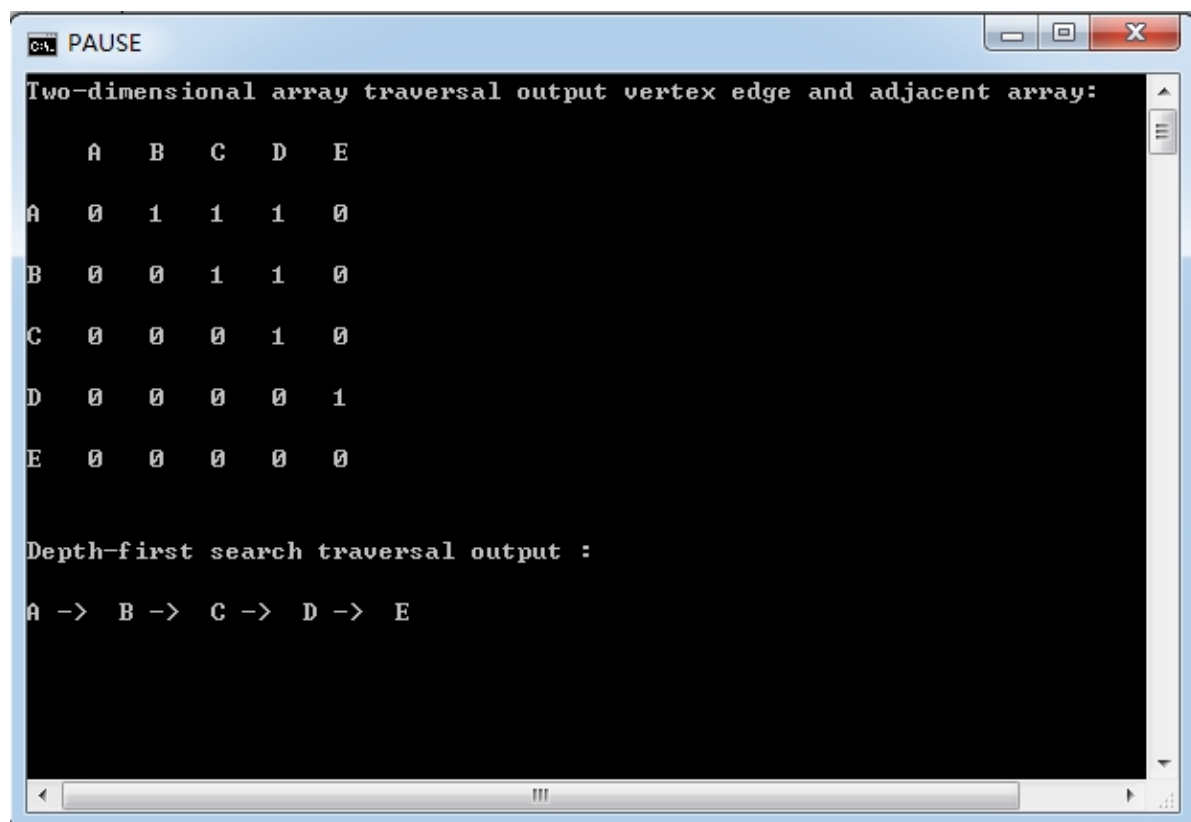
    # Add adjacency edge
    graph. add_edge( 0 , 1 )
    graph. add_edge( 0 , 2 )
    graph. add_edge( 0 , 3 )
    graph. add_edge( 1 , 2 )
    graph. add_edge( 1 , 3 )
    graph. add_edge( 2 , 3 )
    graph. add_edge( 3 , 4 )
```

```
# Two-dimensional array traversal output vertex edge and adjacent
array
print_graph( graph)

# Depth-first search traversal output
print ( "\nDepth-first search traversal output : \n" )
graph.depth_first_search()

if __name__ == "__main__" :
    main()
```

Result:



```
C:\> PAUSE
Two-dimensional array traversal output vertex edge and adjacent array:

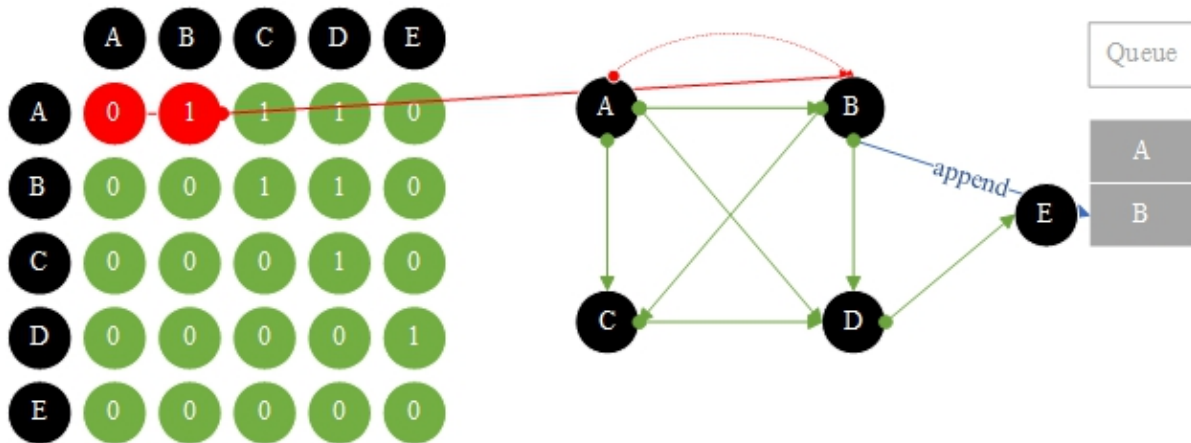
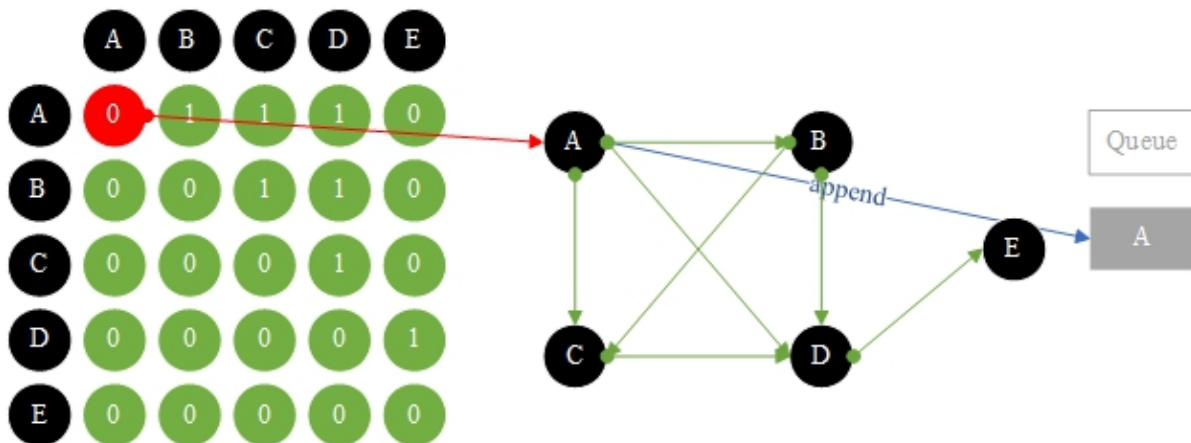
    A    B    C    D    E
A    0    1    1    1    0
B    0    0    1    1    0
C    0    0    0    1    0
D    0    0    0    0    1
E    0    0    0    0    0

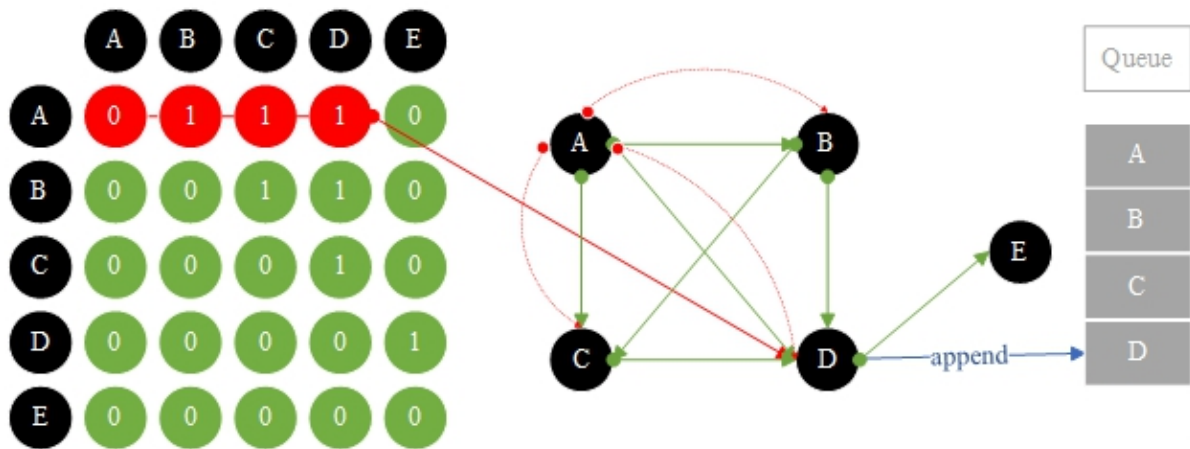
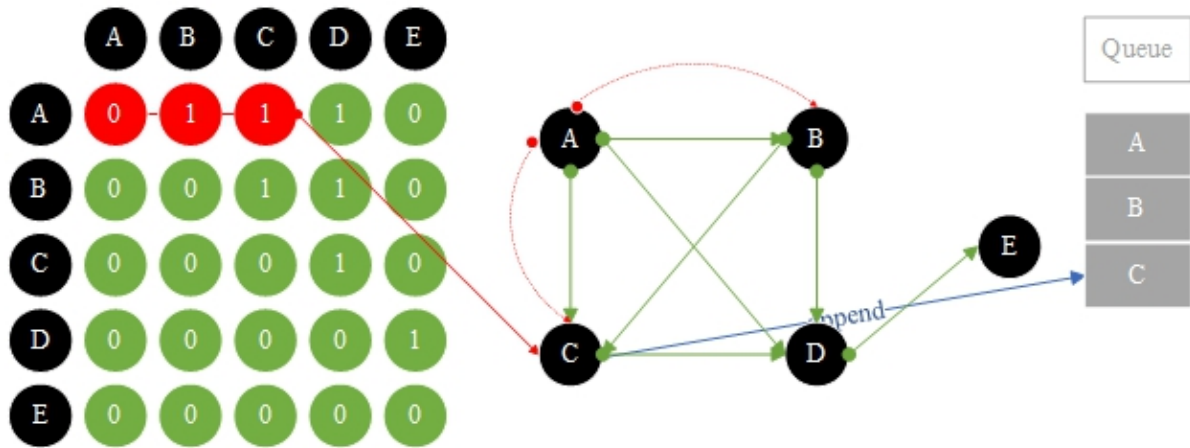
Depth-first search traversal output :
A -> B -> C -> D -> E
```

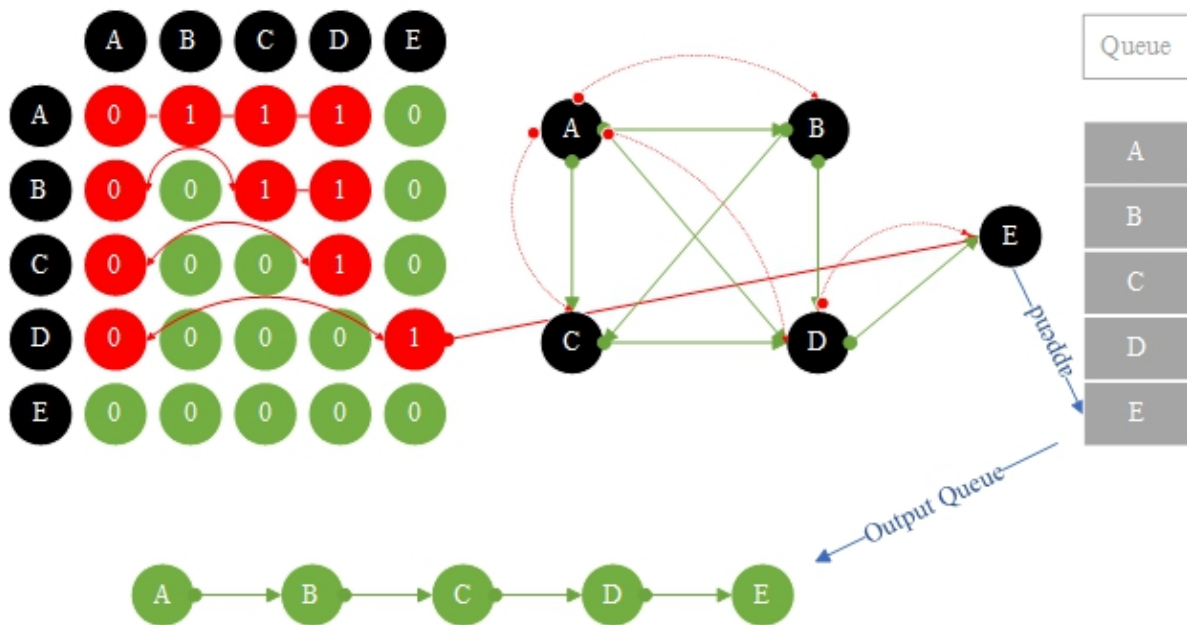
Directed Graph and Breadth-First Search

Breadth-First Search:

Find all neighboring edge nodes B, C, D from A and then find all neighboring nodes A, C, D from B and so on until all nodes are found **A -> B -> C -> D -> E**.







node.py

```
class Node :  
    data = ''  
    prev = None  
    next = None  
  
    def __init__ ( self, data, prev, next):  
        self.data = data  
        self.prev = prev  
        self.next = next
```

vertex.py

```
class Vertex :  
    data = ''  
    visited = False # Have you visited  
  
    def __init__ ( self, data, visited):  
        self.data = data  
        self.visited = visited
```

queue.py

```
from node import Node
class Queue :
    __head = None
    __tail = None
    __count = 0 # Number of elements

    def add ( self, element):
        if self.__head == None :
            self.__head = Node( element, None ,
None )
            self.__tail = self.__head
        else :
            new_node = Node( element, None ,
None )
            self.__tail.next = new_node
            new_node.prev = self.__tail
            self.__tail = new_node
            self.__count += 1

    def poll ( self):
        p = self.__head
        self.__head = self.__head.next

        p.next = None
        p.prev = None

        self.__count -= 1
        return p.data

    def peek ( self):
        return self.__head.data

    def size ( self):
        return self.__count

    def is_empty ( self):
        if self.__count == 0 :
```

```
return True  
else :  
return False
```


graph.py

test_graph.py

```
from graph import Graph

def print_graph ( graph):
    print ( "Two-dimensional array traversal output vertex edge and
adjacent array: \n " )
    length = len( graph.get_vertexs())
    print ( "    ", end= "" )
    for i in range( 0 , length):
        print ( graph.get_vertexs()[ i].data , " " , end= "" )
    print ( "\n" )

    length = len( graph.get_adjacency_matrix())
    for i in range( 0 , length):
        print ( graph.get_vertexs()[ i].data , " " , end= "" )
        for j in range( 0 , length):
            print ( graph.get_adjacency_matrix()[ i][ j] , " " , end= "" )
        print ( "\n" )

def main ():
    graph = Graph( 5 )

    # Add Vertex
    graph.add_vertex( "A" )
    graph.add_vertex( "B" )
    graph.add_vertex( "C" )
    graph.add_vertex( "D" )
    graph.add_vertex( "E" )

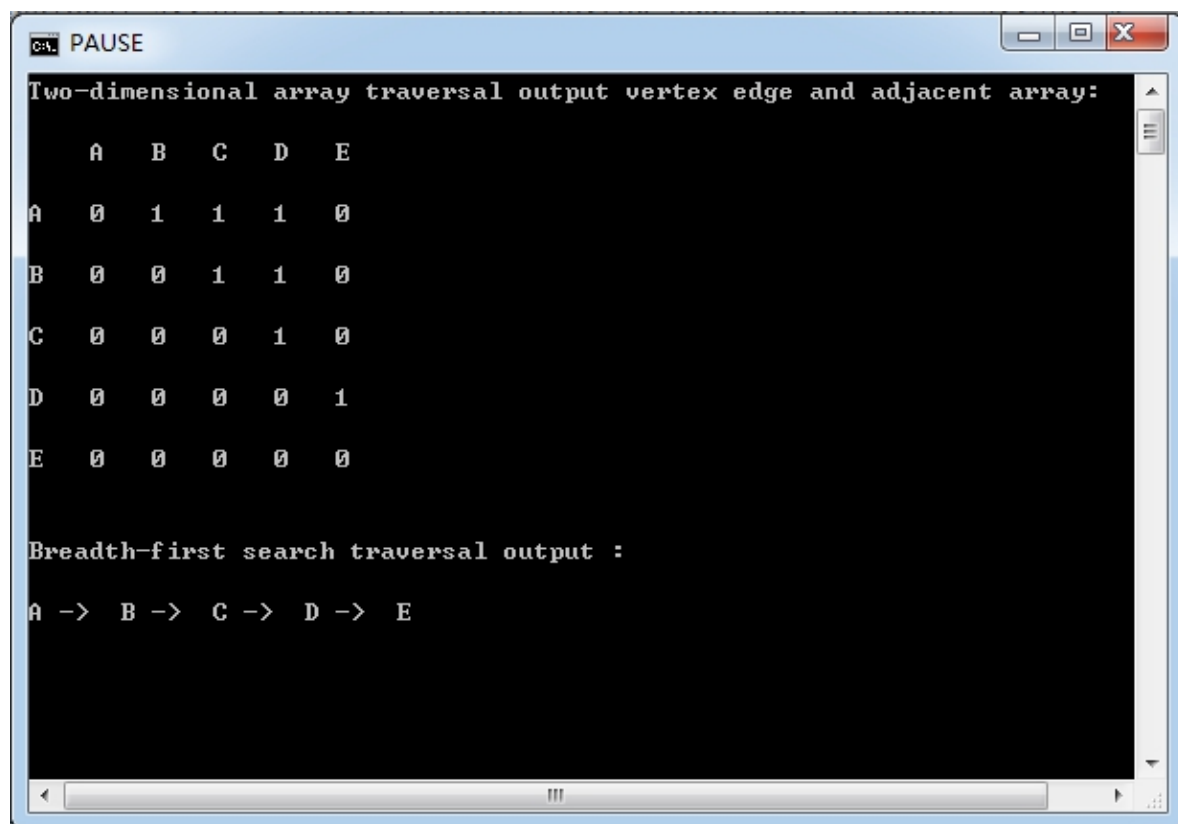
    # Add adjacency edge
    graph.add_edge( 0 , 1 )
    graph.add_edge( 0 , 2 )
    graph.add_edge( 0 , 3 )
    graph.add_edge( 1 , 2 )
    graph.add_edge( 1 , 3 )
    graph.add_edge( 2 , 3 )
    graph.add_edge( 3 , 4 )
```

```
# Two-dimensional array traversal output vertex edge and adjacent
array
print_graph( graph)

# Breadth-first search traversal output
print ( "\nBreadth-first search traversal output : \n" )
graph.breadth_first_search()

if __name__ == "__main__" :
    main()
```

Result:



```
C:\> PAUSE
Two-dimensional array traversal output vertex edge and adjacent array:

   A   B   C   D   E
A   0   1   1   1   0
B   0   0   1   1   0
C   0   0   0   1   0
D   0   0   0   0   1
E   0   0   0   0   0

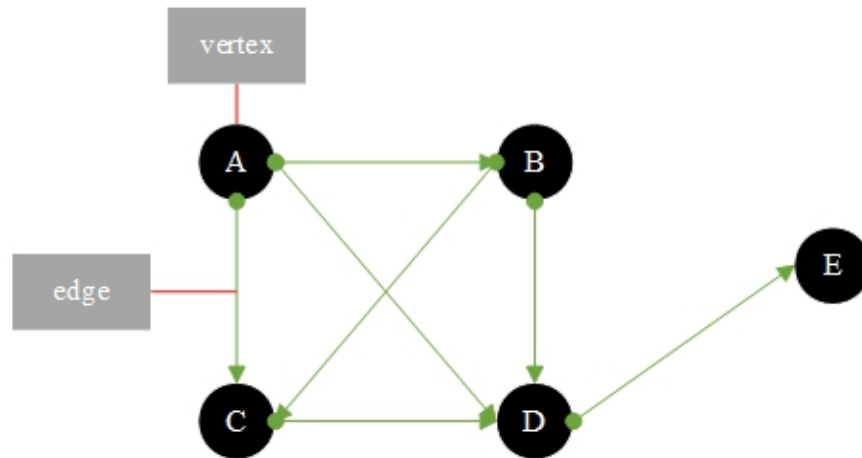
Breadth-first search traversal output :
A ->  B ->  C ->  D ->  E
```

Directed Graph Topological Sorting

Directed Graph Topological Sorting:

Sort the vertices in the directed graph with order of direction

.
Directed Graph has direction : **A -> B** and **B -> A** are different



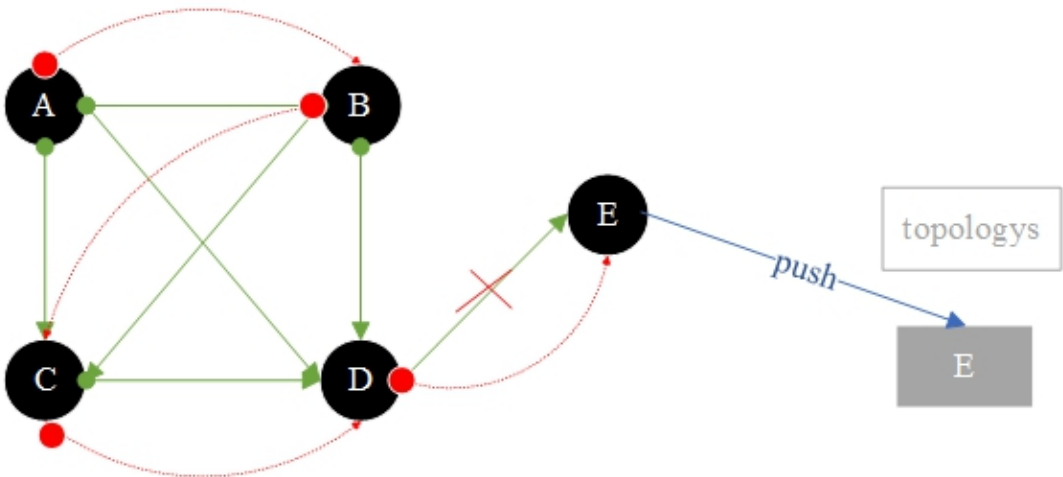
1. The adjacency matrix is described above:

The total number of vertices is a two-dimensional array size, if have value of the edge is **1** , otherwise no value of the edge is **0** .

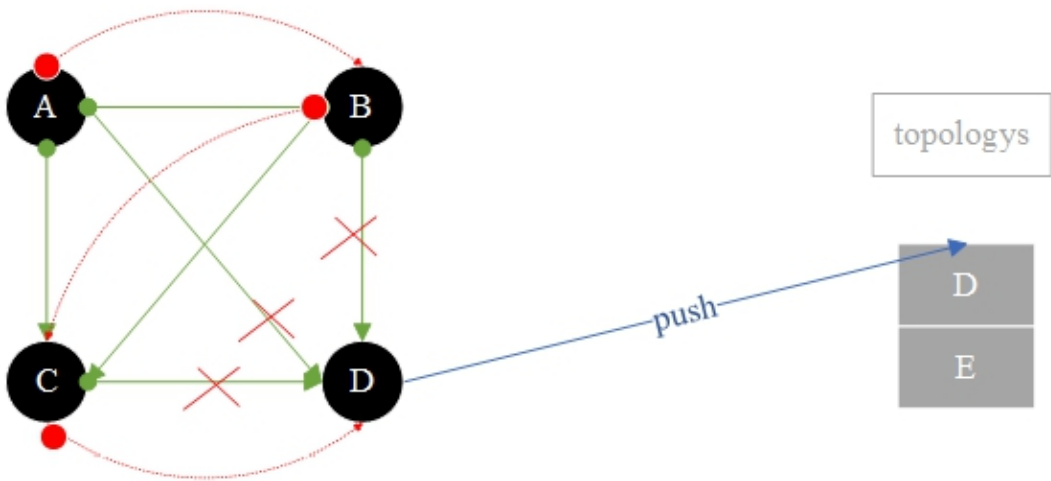
	A	B	C	D	E
A	0	1	1	1	0
B	0	0	1	1	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	0	0	0

Topological sorting from vertex A : **A -> B -> C -> D -> E**

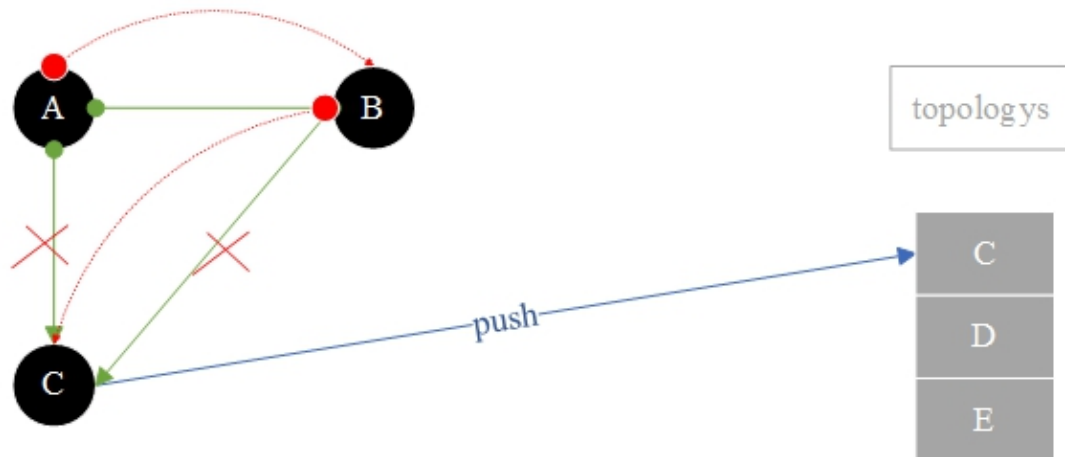
Find no successor vertices E then save to topologys, last E remove from the graph



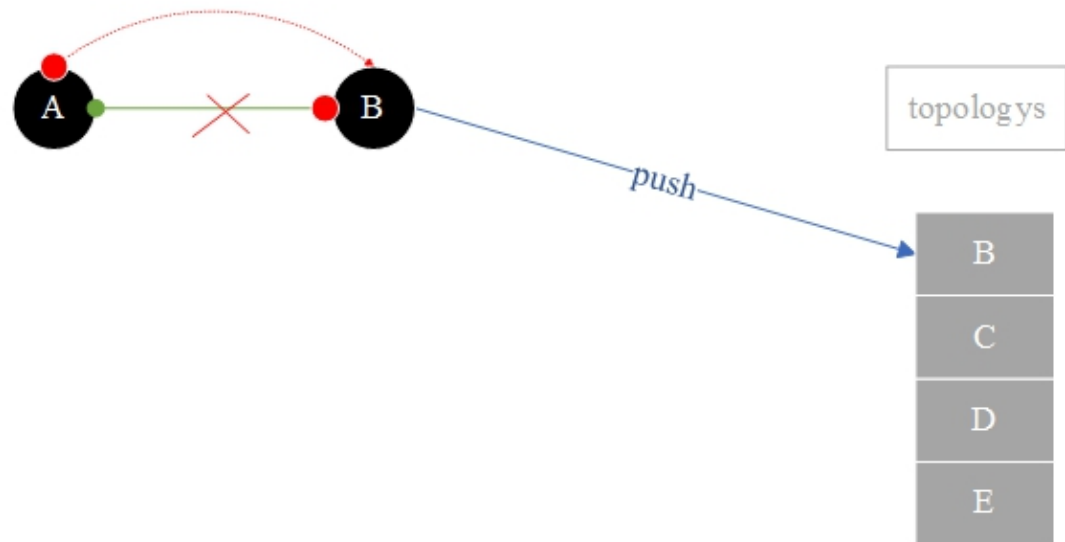
Find no successor vertices D then save to topologys, last D remove from the graph



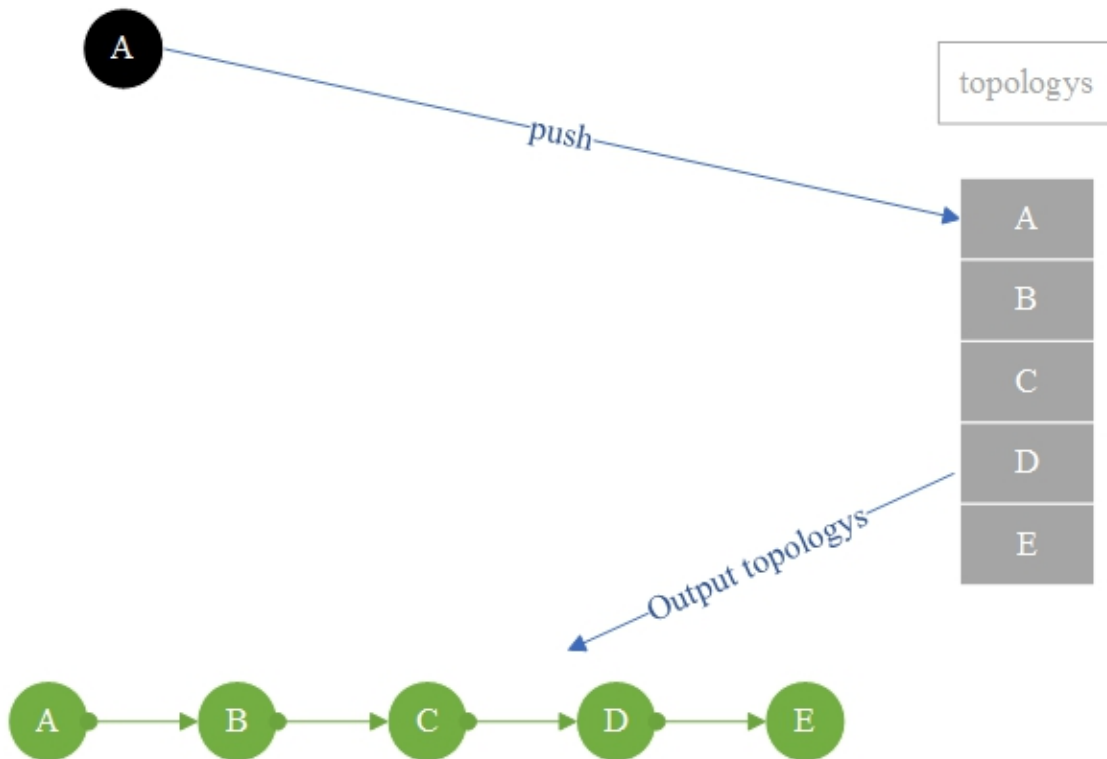
Find no successor vertices C then save to topologys, last C remove from the graph



Find no successor vertices C then save to topologys, last C remove from the graph



Find no successor vertices C then save to topologys, last C remove from the graph



node.py

```
class Node :  
    data = ''  
    prev = None  
    next = None  
  
    def __init__ ( self, data, prev, next):  
        self.data = data  
        self.prev = prev  
        self.next = next
```

vertex.py

```
class Vertex :  
    data = ''  
    visited = False # Have you visited  
  
    def __init__ ( self, data, visited):  
        self.data = data  
        self.visited = visited
```


queue.py

```
from node import Node

class Queue :
    __head = None
    __tail = None
    __count = 0 # Number of elements

    def add ( self, element):
        if self.__head == None :
            self.__head = Node( element, None ,
None )
            self.__tail = self.__head
        else :
            new_node = Node( element, None ,
None )
            self.__tail.next = new_node
            new_node.prev = self.__tail
            self.__tail = new_node
            self.__count += 1

    def poll ( self):
        p = self.__head
        self.__head = self.__head.next

        p.next = None
        p.prev = None

        self.__count -= 1
        return p.data

    def peek ( self):
        return self.__head.data

    def size ( self):
        return self.__count

    def is_empty ( self):
```

```
if self.__count == 0 :  
    return True  
else :  
    return False
```

graph.py

test_topology.py

```
from graph import Graph

def print_graph ( graph):
    print ( "Two-dimensional array traversal output vertex edge and
adjacent array: \n " )
    length = len( graph. get_vertexs())
    print ( "    ", end= "" )
    for i in range( 0 , length):
        print ( graph. get_vertexs()[ i]. data , " " , end= "" )
    print ( "\n" )

    length = len( graph. get_adjacency_matrix())
    for i in range( 0 , length):
        print ( graph. get_vertexs()[ i]. data , " " , end= "" )
        for j in range( 0 , length):
            print ( graph. get_adjacency_matrix()[ i][ j] , " " , end= "" )
        print ( "\n" )

def main ():
    graph = Graph( 5 )

    # Add Vertex
    graph. add_vertex( "A" )
    graph. add_vertex( "B" )
    graph. add_vertex( "C" )
    graph. add_vertex( "D" )
    graph. add_vertex( "E" )

    # Add adjacency edge
    graph. add_edge( 0 , 1 )
    graph. add_edge( 0 , 2 )
    graph. add_edge( 0 , 3 )
    graph. add_edge( 1 , 2 )
    graph. add_edge( 1 , 3 )
    graph. add_edge( 2 , 3 )
    graph. add_edge( 3 , 4 )
```

```
# Two-dimensional array traversal output vertex edge and adjacent  
array
```

```
print_graph( graph)
```

```
print ( "Directed graph topological ordering:" )
```

```
graph. topology_sort()
```

```
length = len( graph. get_topologys())
```

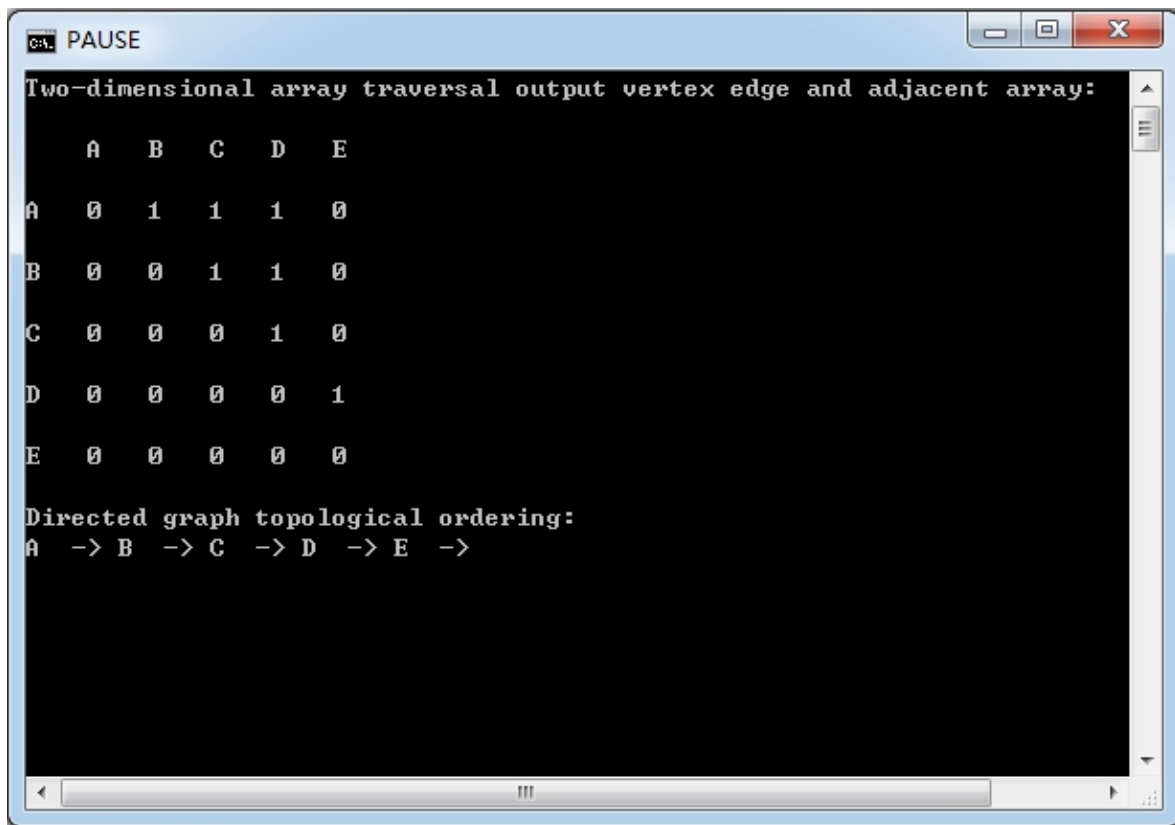
```
for i in range( 0 , length):
```

```
    print ( graph. get_topologys()[ i]. data, " -> " , end= "" )
```

```
if __name__ == "__main__" :
```

```
    main()
```

Result:



```
Two-dimensional array traversal output vertex edge and adjacent array:
```

	A	B	C	D	E
A	0	1	1	1	0
B	0	0	1	1	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	0	0	0

```
Directed graph topological ordering:  
A -> B -> C -> D -> E ->
```

Towers of Hanoi

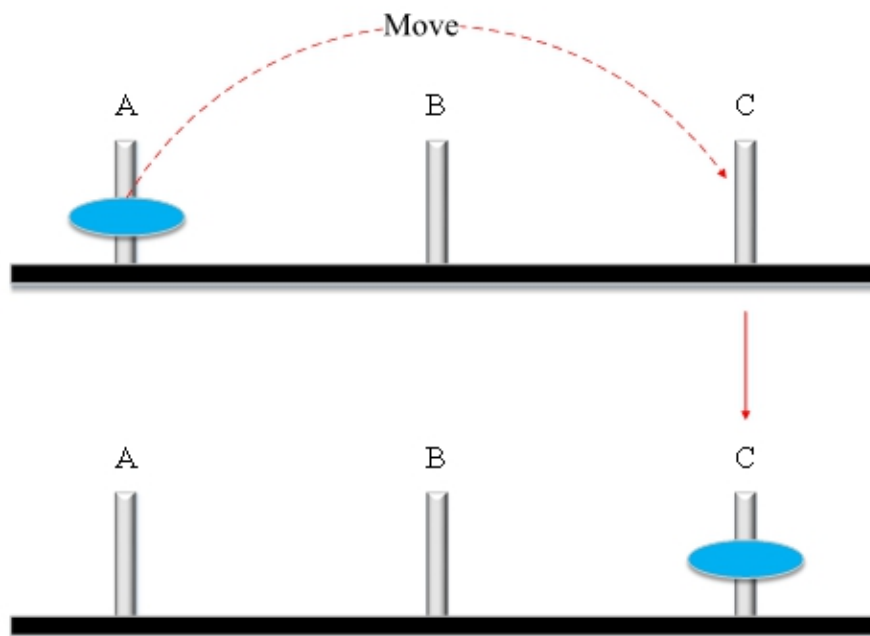
The Hanoi Tower that a Frenchman M. Claus (Lucas) from Thailand to France in 1883. Hanoi Tower which is supported by three diamond Pillars. At the beginning, God placed 64 gold discs from top to bottom on the first Pillar. God ordered the monks to move all gold discs from the first Pillar to the third Pillar. The principle of large plates under small plates during the handling process. If only one plate is moved daily, the tower will be destroyed. when all the discs are moved that is the end of the world.

Let's turn this story into an algorithm:

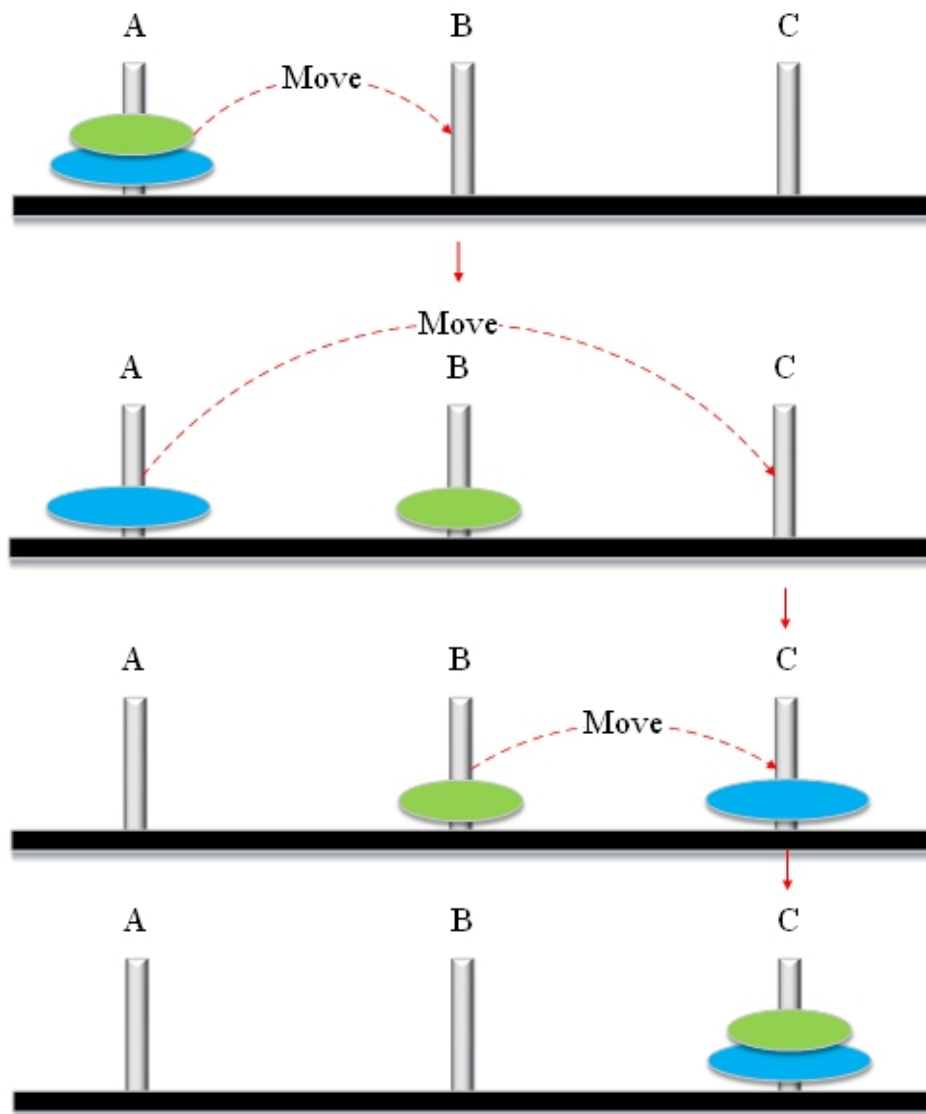
Mark the three columns as ABC.

1. If there is only one disc, move it directly to C (**A->C**).
2. When there are two discs, use B as an auxiliary (**A->B, A->C, B->C**).
3. If there are more than two discs, use B as an auxiliary(**A->B, A->C, B->C**), and continue to recursive process.

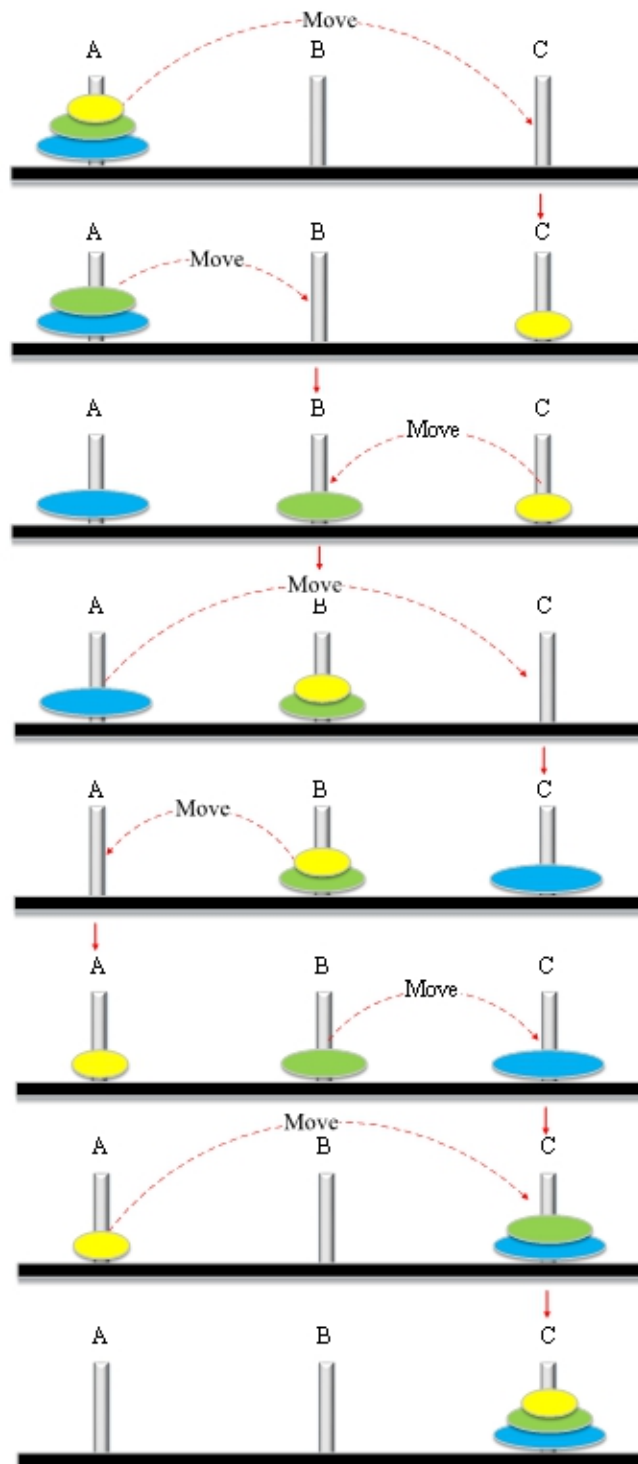
1. If there is only one disc, move it directly to C (**A->C).**



2. When there are two discs, use B as an auxiliary ($A \rightarrow B$, $A \rightarrow C$, $B \rightarrow C$).



3. If more than two discs, use B as an auxiliary, and continue to recursive process.



TowersOfHanoi.py

```
import sys

def hanoi ( n, A, B, C):
    if ( n == 1 ):
        print ( "Move " + str( n) + " " + A + " to " + C + "\n" )
    else :
        hanoi( n - 1 , A, C, B) # Move the n-1th disc on the A through C
to B
        print ( "Move " + str( n) + " from " + A + " to " + C + "\n" )
        hanoi( n - 1 , B, A, C) #Move the n-1th disc on the B through A
to C

def main ():
    n= int( input( "Please enter the number of discs : \n" ))
    hanoi( n, "A" , "B" , "C" )

if __name__ == "__main__" :
    main()
```

Result:

Please enter the number of discs :

1

Move 1 A to C

Please enter the number of discs :

2

Move 1 A to B

Move 2 from A to C

Move 1 B to C

Please enter the number of discs :

3

Move 1 A to C

Move 2 from A to B

Move 1 C to B

Move 3 from A to C

Move 1 B to A

Move 2 from B to C

Move 1 A to C

Fibonacci

Fibonacci : a European mathematician in the 1200s, in his writings: "If there is a rabbit

After a month can birth to a newborn rabbit. At first there was only 1 rabbit, after one month still 1 rabbit. after two month 2 rabbit, and after three months there are 3 rabbit

for example: 1, 1, 2, 3, 5, 8, 13 ...

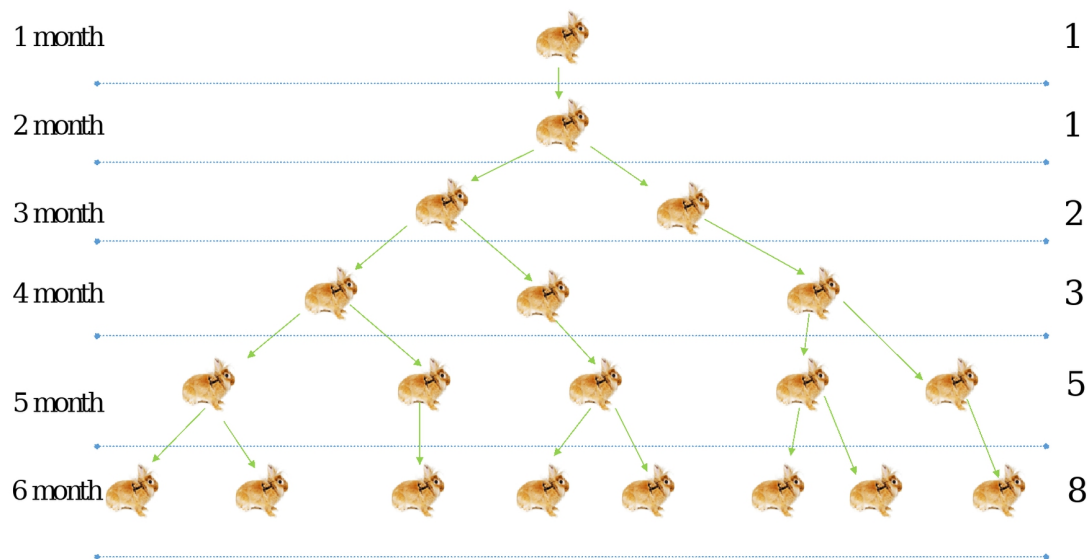
Fibonacci definition:

if $n = 0, 1$

$fn = n$

if $n > 1$

$fn = fn-1 + fn-2$



Fibonacci.py

```
import sys

def fibonacci ( n):
    if ( n == 1 or n == 2 ):
        return 1
    else :
        return fibonacci( n - 1 ) + fibonacci( n - 2 )

def main ():
    number= int( input( "Please enter the number of month : \n" ))
    for i in range( 1 , number):
        print ( str( i) + " month : " + str( fibonacci( i)) + "\n" )

if __name__ == "__main__" :
    main()
```

Result:

Please enter the number of month :

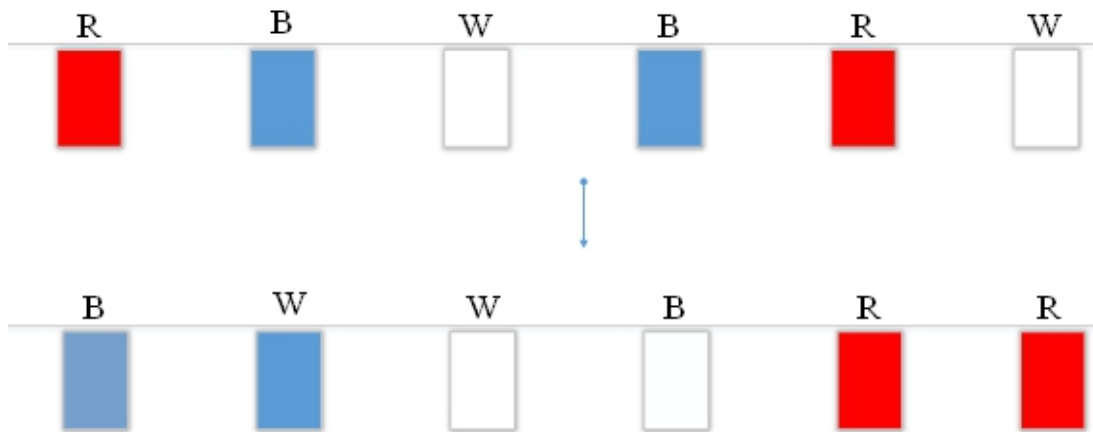
7

1 month : 1
2 month : 1
3 month : 2
4 month : 3
5 month : 5
6 month : 8

Dijkstra

The tricolor flag was originally raised by E.W. Dijkstra, who used the Dutch national flag (Dijkstra is Dutch).

Suppose there is a rope with red, white, and blue flags. At first all the flags on the rope are not in order. You need to arrange them in the order of **blue - > white -> red**. How to move them with the least times. you just only do this on the rope, and only swap two flags at a time.



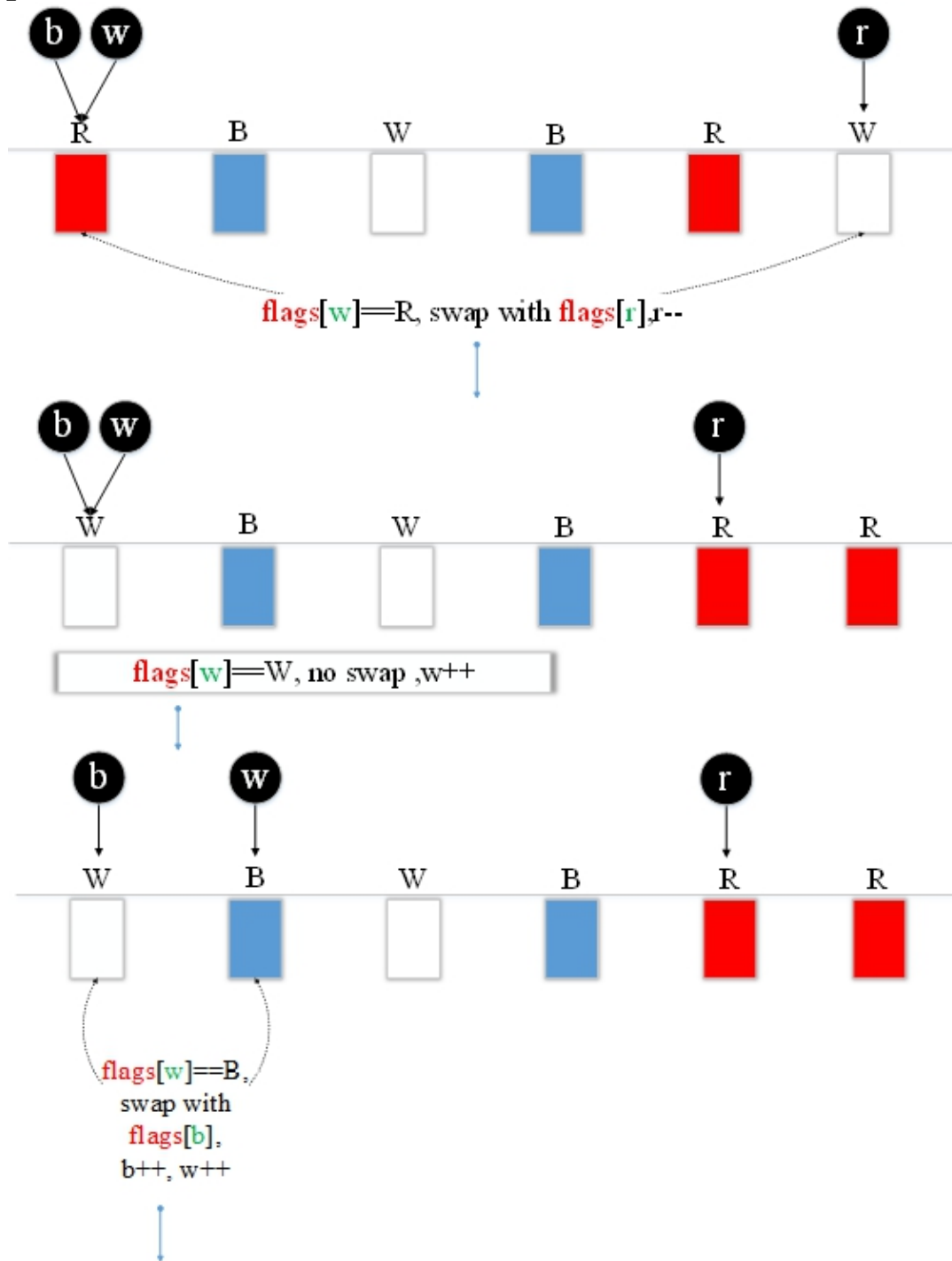
Solution:

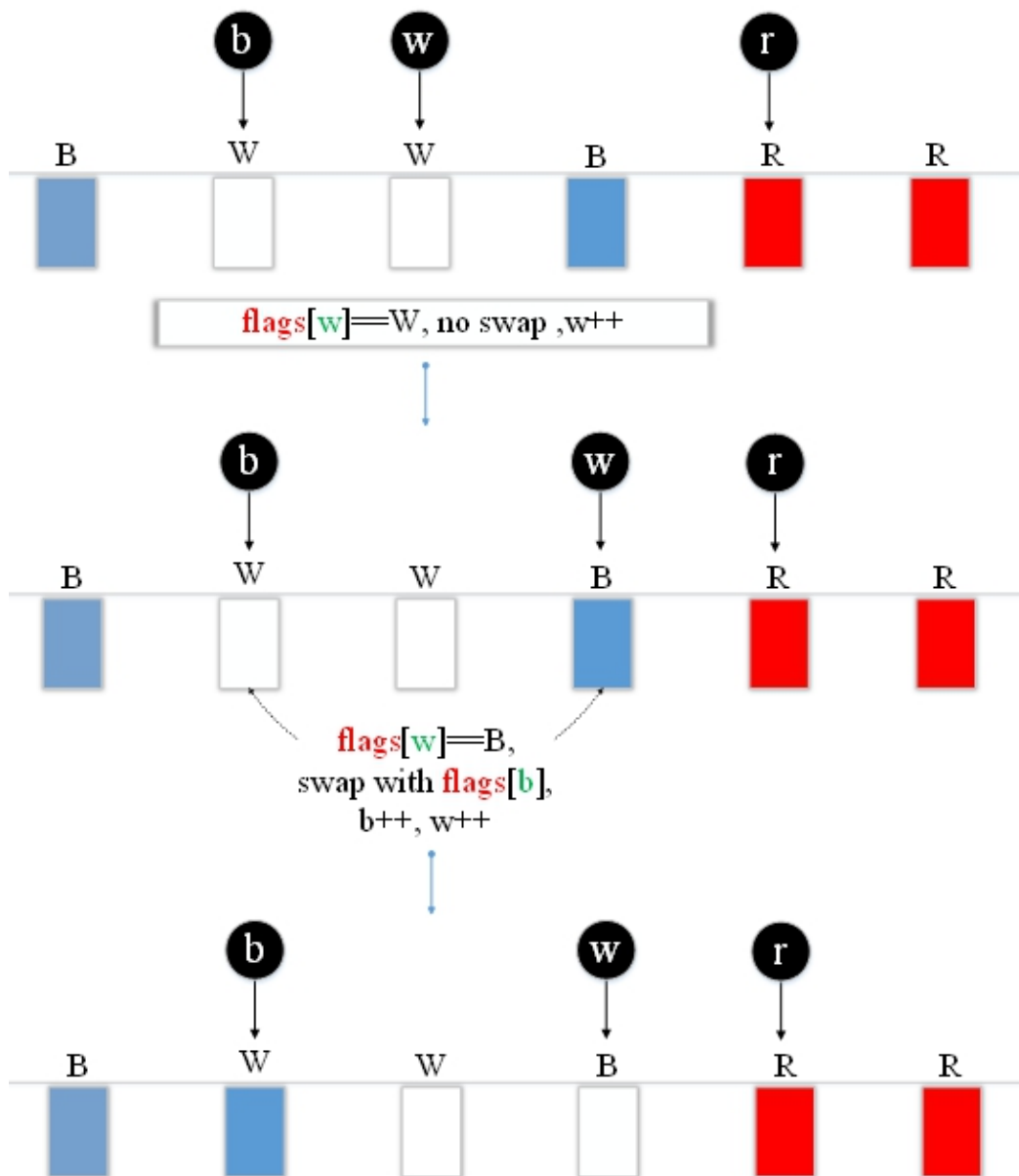
Use the **char arrays** to store the **flags**. For example, **b**, **w**, and **r** indicate the position of **blue**, **white** and **red** flags. The beginning of **b** and **w** is 0 of the array, and **r** is at the end of the array.

- (1) If the position of **w** is a blue flag, **flags[w]** exchange with **flags[b]**. And **whiteIndex** and **b** is moved backward by 1.
- (2) If the position of **w** is a white flag, **w** moves backward by 1.
- (3) If the position of **w** is a red flag, **flags[w]** exchange with **flags[r]**. **r** moves forward by 1.

In the end, the flags in front of **b** are all blue, and the flags behind **r** are all red.

Graphic Solution





Dijkstra.py

```
import sys

def main():
    flags = [ 'R' , 'B' , 'W' , 'B' , 'R' , 'W' ]
    length = len( flags )
    b = 0
    w = 0
    r = length - 1
    count = 0
    while ( w <= r ):
        if ( flags[ w ] == 'W' ):
            w = w + 1
        elif ( flags[ w ] == 'B' ):
            temp = flags[ w ]
            flags[ w ] = flags[ b ]
            flags[ b ] = temp
            w = w + 1
            b = b + 1
            count = count + 1
        elif ( flags[ w ] == 'R' ):
            m = flags[ w ]
            flags[ w ] = flags[ r ]
            flags[ r ] = m
            r = r - 1
            count = count + 1

    for i in range( 0 , length ):
        print ( flags[ i ], end= "" )
    print ( "\nThe total exchange count : " + str( count ))

if __name__ == "__main__" :
    main()
```

Result:

BBWWRR

The total exchange count : 4

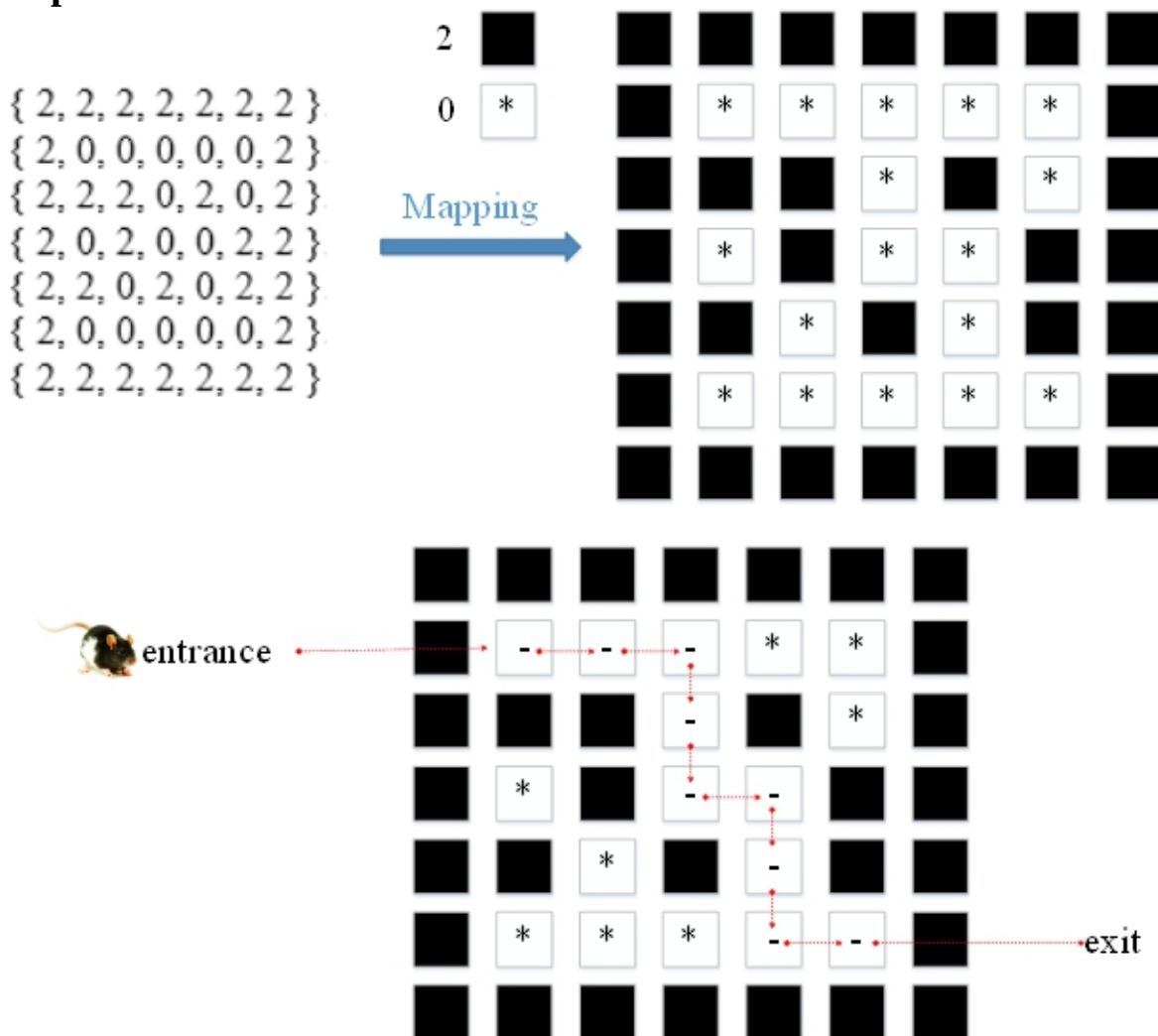
Mouse Walking Maze

Mouse Walking Maze is a basic type of recursive solution. We use **2** to represent the **wall** in a **two-dimensional array** , and use **1** to represent the **path** of the mouse, and try to find the path from the entrance to the exit.

Solution:

The mouse moves in four directions: **up, left, down, and right** . if hit the **wall** go back and select the next forward direction, so test the four directions in the array until mouse reach the exit.

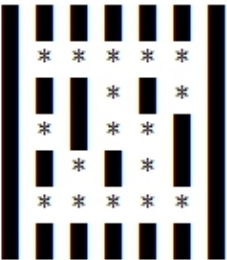
Graphic Solution



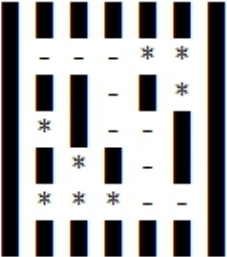
MouseWalkingMaze.py

Result:

Maze:



Maze Path :



Eight Coins

There are eight coins with the same appearance, one is a counterfeit coin, and the weight of counterfeit coin is different from the real coin, but it is unknown whether the counterfeit coin is lighter or heavier than the real coin. Please design a efficient algorithm to detect this counterfeit coin.

Solution:

Take six **a, b, c, d, e, f** from eight coins, and put three to the balance for comparison. Suppose **a, b, c** are placed on one side, and **d, e, f** are placed on the other side.

1. $a + b + c > d + e + f$
2. $a + b + c = d + e + f$
3. $a + b + c < d + e + f$

If $a + b + c > d + e + f$, there is a counterfeit coin in one of the six coins, and **g, h** are real coins. At this time, one coin can be removed from both side. Suppose that **c and f** are removed. At the same time, one coin at each side is replaced. Suppose the coins **b and e** are interchanged, and then the second comparison. There are also three possibilities:

1. $a + e > d + b$: the counterfeit currency must be one of **a, d**. as long as we compare a real currency **h** with **a**, we can find the counterfeit currency. If $a > h$, **a** is a heavier counterfeit currency; if $a = h$, **d** is a lighter counterfeit currency.

2. $a + e = d + b$: the counterfeit currency must be one of **c, f**, and the real coin **h** is compared with **c**. If $c > h$, **c** is a heavier counterfeit currency; if $c = h$, then **f** is a lighter counterfeit currency.

3. $a + e < d + b$: one of **b or e** is a counterfeit coin, Also use the real coin **h** to compare with **b**, if $b > h$, then **b** is a heavier counterfeit currency; if $b = h$, then **e** is a lighter counterfeit currency;

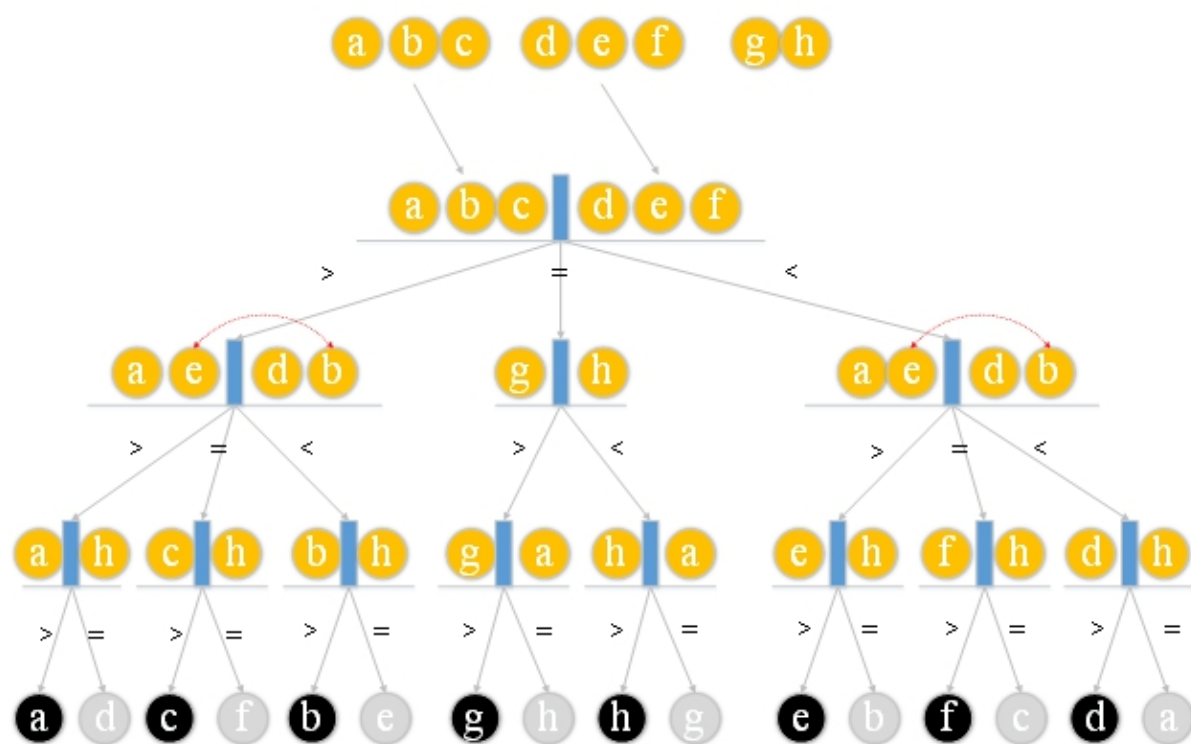
Graphic Solution



Heavy



Light



EightCoins.py

```
import sys
import random

def compare ( coins, i, j, k): #coin[k] true,coin[i]>coin[j]
    if ( coins[ i ] > coins[ k]): #coin[i]>coin[j]&&coin[i]>coin[k] -----
        >coin[i] is a heavy counterfeit coin
        print ( "\nCounterfeit currency " + str(( i + 1 )) + " is heavier " )
    else : #coin[j] is a light counterfeit coin
        print ( "\nCounterfeit currency " + str(( j + 1 )) + " is lighter " )

def eightcoins ( coins):
    if ( coins[ 0 ] + coins[ 1 ] + coins[ 2 ] == coins[ 3 ] + coins[ 4 ] +
        coins[ 5 ]): #(a+b+c)==(d+e+f)
        if ( coins[ 6 ] > coins[ 7 ]): #g>h?(g>a?g:a):(h>a?h:a)
            compare( coins, 6 , 7 , 0 )
        else : #h>g?(h>a?h:a):(g>a?g:a)
            compare( coins, 7 , 6 , 0 )
        elif ( coins[ 0 ] + coins[ 1 ] + coins[ 2 ] > coins[ 3 ] + coins[ 4 ] +
            coins[ 5 ]): #(a+b+c)>(d+e+f)
            if ( coins[ 0 ] + coins[ 3 ] == coins[ 1 ] + coins[ 4 ]): #(a+e)==
            (d+b)
                compare( coins, 2 , 5 , 0 )
            elif ( coins[ 0 ] + coins[ 3 ] > coins[ 1 ] + coins[ 4 ]): #(a+e)>
            (d+b)
                compare( coins, 0 , 4 , 1 )
            if ( coins[ 0 ] + coins[ 3 ] < coins[ 1 ] + coins[ 4 ]): #(a+e)<(d+b)
                compare( coins, 1 , 3 , 0 )
            elif ( coins[ 0 ] + coins[ 1 ] + coins[ 2 ] < coins[ 3 ] + coins[ 4 ] +
                coins[ 5 ]): #(a+b+c)<(d+e+f)
                if ( coins[ 0 ] + coins[ 3 ] == coins[ 1 ] + coins[ 4 ]): #(a+e)>(d+b)
                    compare( coins, 5 , 2 , 0 )
                elif ( coins[ 0 ] + coins[ 3 ] > coins[ 1 ] + coins[ 4 ]): #(a+e)>
                (d+b)
                    compare( coins, 3 , 1 , 0 )
                if ( coins[ 0 ] + coins[ 3 ] < coins[ 1 ] + coins[ 4 ]): #(a+e)<(d+b)
```

```
compare( coins, 4 , 0 , 1 )
```

```
def main ():
```

```
    # Initial coin weight is 10
```

```
    coins = [ 10 for i in range( 8 )]
```

```
    coins[ random.randint( 0 , 7 )] = int( input( "Enter weight of  
counterfeit currency (larger or smaller than 10) : \n" ))
```

```
    eightcoins( coins)
```

```
    for i in range( 0 , 8 ):
```

```
        print ( str( coins[ i] ) + " , " , end= "" )
```

```
if __name__ == "__main__" :
```

```
    main()
```

Result:

First run:

Enter weight of counterfeit currency (larger or smaller than 10) :

2

Counterfeit currency 2 is lighter

10 , 2 , 10 , 10 , 10 , 10 , 10 , 10 ,

Run again:

Enter weight of counterfeit currency (larger or smaller than 10) :

13

Counterfeit currency 4 is heavier

10 , 10 , 10 , 13 , 10 , 10 , 10 , 10 ,

Knapsack Problem

Suppose you have a backpack with a weight of up to 8 kg, and you want to the backpack with a total price of Products, suppose the fruit (**ID**, **Name**, **Price** and **Weight**)

ID	Name	Price	Weight
0	Plum	4kg	4500
1	Apple	5kg	5700
2	Orange	2kg	2250
3	Strawberry	1kg	1100
4	Melon	6kg	6700

Solution:

To solve the optimization problem we can use **Dynamic Programming** . In the beginning there is a empty set, every time add an element, find the best solution at this stage, until all elements are added. After entering the set finally can get the best solution.

There are two arrays, **value** and **item**

value: the total price of the current best solution.

item : the last fruit in the backpack.

there are 8 backpacks with a weight of 1 to 8, and find the best solution for each backpack.

Gradually put the fruit in the backpack and find the best solution:

1. Put in plums:

Weight of Backpack	1kg	2 k g	3 kg	4 kg	5 kg	6 kg	7 kg	8 kg
Value	0	0	0	4500	4500	4500	4500	9000
Item	-	-	-	0	0	0	0	0 , 0
Name	-	-	-	Plum	Plum	Plum	Plum	2* Plum

2. Put in apples:

Weight of Backpack	1kg	2 kg	3 kg	4 kg	5 kg	6 kg	7 kg	8 kg
Value	0	0	0	4500	5700	5700	5700	9000
Item	-	-	-	0	1	1	1	0 , 0
Name	-	-	-	Plum	Apple	Apple	Apple	2* Plum

3. Put in oranges:

Weight of Backpack	1 kg	2 kg	3 kg	4 kg	5 kg	6 kg	7 kg	8 kg
Value	0	2250	2250	4500	5700	6750	7950	9000
Item	-	2	2	0	1	0 , 2	1 , 2	0 , 0
Name	-	Orange	Orange	Plum	Apple	Orange Plum	Orange Apple	2* Plum

4. Put in strawberries:

5. Put in melons:

Weight of Backpack	1kg	2 kg	3 kg	4 kg	5 kg	6 kg	7 kg	8 kg
Value	1100	2250	3350	4500	5700	6800	7950	9050
Item	3	2	2 , 3	0	1	0 , 3	1 , 2	1 , 2, 3
Name	Strawberry	Orange	Orange Strawberry	Plum	Apple	Plum Strawberry	Apple Orange	Apple Orange Strawberry

From the last table that when the backpack weighs 8 kg, a maximum of 9050, so the best solution is to put **Strawberries, Oranges and Apples** , and the total price is 9050.

Knapsack.py

```
import sys

def main():
    MAXSIZE = 8 ;
    MINSIZE = 1 ;
    item = [ 0 for _ in range( MAXSIZE + 1 )]
    value = [ 0 for _ in range( MAXSIZE + 1 )]
    fruits = [
        Fruit( "Plum " , 4 , 4500 ),
        Fruit( "Apple" , 5 , 5700 ),
        Fruit( "Orange" , 2 , 2250 ),
        Fruit( "Strawberry" , 1 , 1100 ),
        Fruit( "Melon" , 6 , 6700 ) ]

    length = len( fruits)
    for i in range( 0 , length):
        size = fruits[ i]. getSize()
        for j in range( size, MAXSIZE+ 1 ):
            p = j - fruits[ i]. getSize()
            newValue = value[ p] + fruits[ i]. getPrice()
            if ( newValue > value[ j]): # Find the best solution
                value[ j] = newValue
                item[ j] = i

    print ( "Item \t Price" )
    i = MAXSIZE
    while ( i >= MINSIZE):
        print ( fruits[ item[ i]]. getName() + "\t" + str( fruits[ item[ i]].
getPrice()))
        i = i - fruits[ item[ i]]. getSize()
    print ( "Total \t" + str( value[ MAXSIZE]))
```

```
class Fruit :  
    def __init__ ( self, name, size, price):  
        self.name = name  
        self.size = size  
        self.price = price  
  
    def getName ( self):  
        return self.name  
  
    def getPrice ( self):  
        return self.price  
  
    def getSize ( self):  
        return self.size  
  
if __name__ == "__main__" :  
    main()
```

Result:

Item	Price
Strawberry	1100
Orange	2250
Apple	5700
Total	9050

Josephus Problem

There are 9 Jewish hid in a hole with Josephus and his friends . The 9 Jews decided to die rather than be caught by the enemy, so they decided In a suicide method, 11 people are arranged in a circle, and the first person reports the number. After each number is reported to the third person, the person must commit suicide. Then count again from the next one until everyone commits suicide. But Josephus and his friends did not want to obey. Josephus asked his friends to pretend to obey, and he arranged the friends with himself. In the 2th and 7st positions, they escaped this death game.

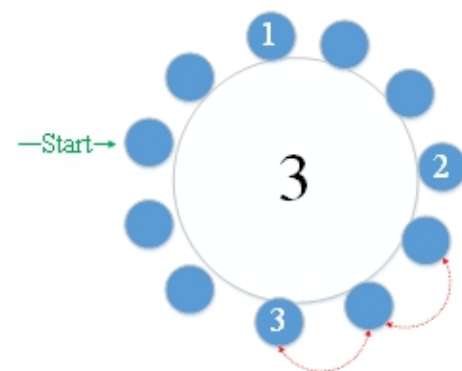
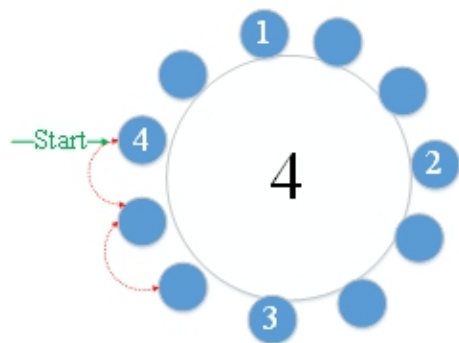
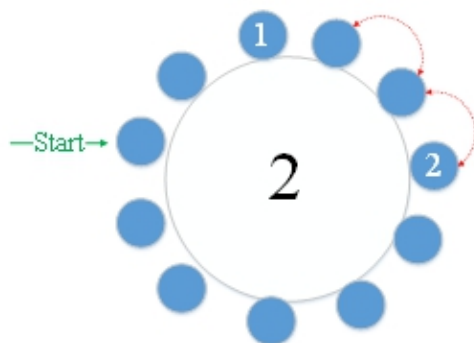
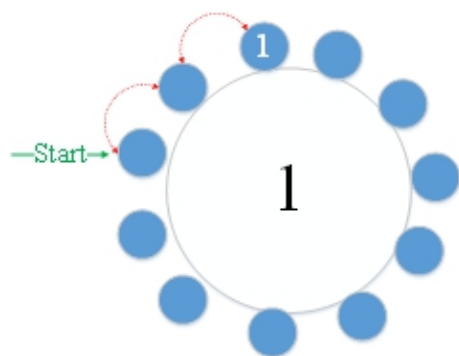
Solution:

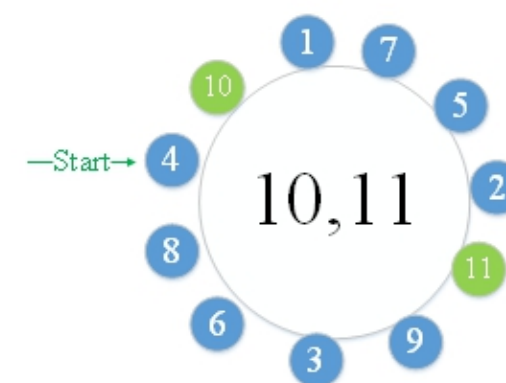
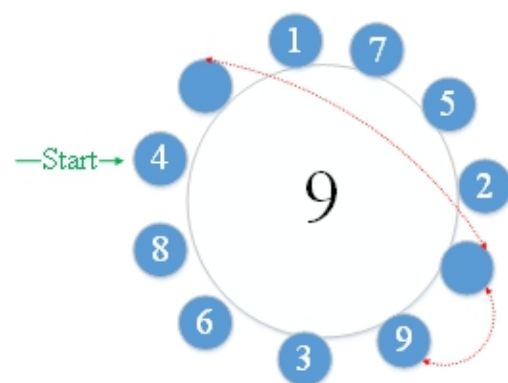
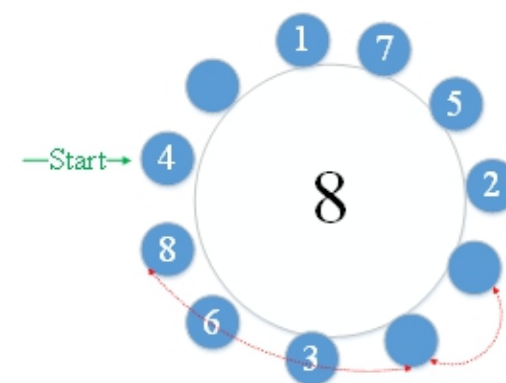
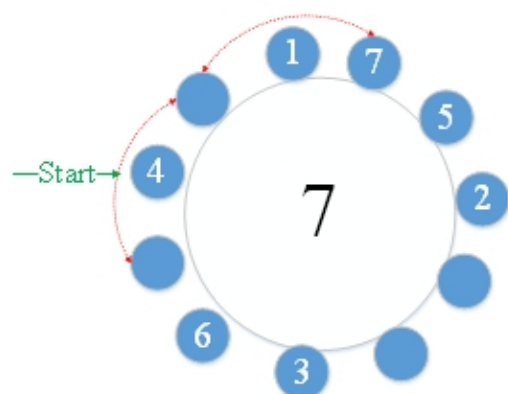
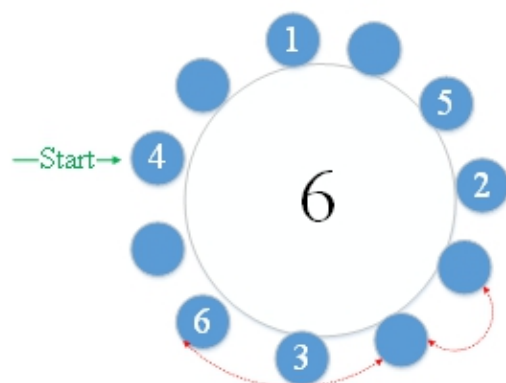
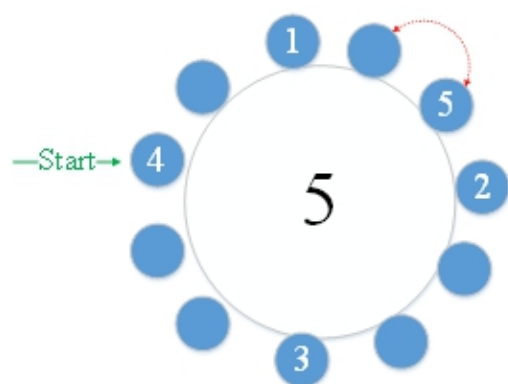
As long as the array is treated as a ring. Fill in a count for each dataless area, until the count reaches 11, and then list the array from index 1, you can know that each suicide order in this position is the Joseph's position. The 11-person position is as follows:

4 10 1 7 5 2 11 9 3 6 8

From the above, the last two suicide was in the 31st and 16th position. The previous one

Everyone died, so they didn't know that Joseph and his friends didn't follow the rules of the game.





Joseph.py

```
import sys

def main():
    N = 11
    M = 3
    man = [ 0 for _ in range( N)]
    count = 1
    i = 0
    pos = - 1
    alive = 0
    while ( count <= N):
        while ( True ):
            pos = ( pos + 1 ) % N; # Ring
            if ( man[ pos] == 0 ):
                i = i + 1
            if ( i == M):
                i = 0
                break
            man[ pos] = count
            count = count + 1

    print ( "\nJoseph sequence : " )
    for i in range( 0 , N):
        print ( str( man[ i]) + " , " , end= "" )

if __name__ == "__main__" :
    main()
```

Result:

Joseph sequence :
4 , 10 , 1 , 7 , 5 , 2 , 11 , 9 , 3 , 6 , 8 ,

If you enjoyed this book and found some benefit in reading this, I'd like to hear from you and hope that you could take some time to post a review on Amazon. Your feedback and support will help us to greatly improve in future and make this book even better.

You can follow this link now.

<http://www.amazon.com/review/create-review?&asin=B08CCZBNXZ>

Different country reviews only need to modify the amazon domain name in the link:

www.amazon.co.uk

www.amazon.de

www.amazon.fr

www.amazon.es

www.amazon.it

www.amazon.ca

www.amazon.nl

www.amazon.in

www.amazon.co.jp

www.amazon.com.br

www.amazon.com.mx

www.amazon.com.au

I wish you all the best in your future success!