



# Understanding Treasury Futures

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Treasury Bond futures were introduced on the Chicago Board of Trade in 1977. The Treasury futures product line has been augmented over the years by the introduction of Ultra 10-year, 10-year, 5-year, 2-year Treasury note and Ultra Treasury bond futures.<sup>1</sup>

This product line has experienced tremendous success as the scale and global significance of U.S. Treasury investment has grown over the years. Today, these products are utilized on an international basis by institutional and individual investors for purposes of both abating and assuming risk exposures.

This document is intended to provide an overview of the fundamentals of trading U.S. Treasury bond and note futures. We assume only a cursory knowledge of coupon-bearing Treasury securities. Thus, we begin with a primer on the operation of cash Treasury markets before moving on to provide some detail regarding the features of the U.S. Treasury futures contracts as well as a discussion of risk management applications with U.S. Treasury futures.

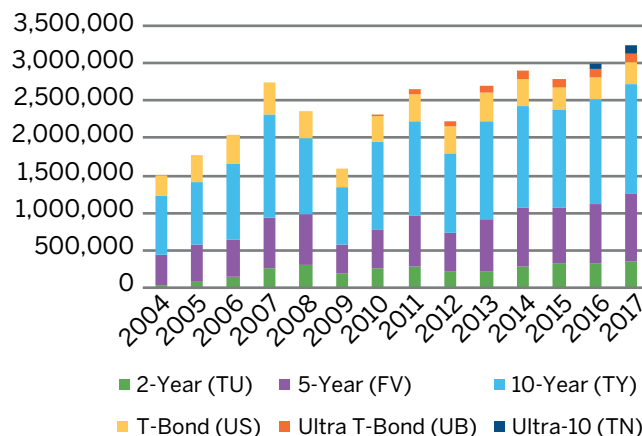
## Coupon-Bearing Treasury Securities

U.S. Treasury bonds and notes represent a loan to the U.S. government. Bondholders are creditors rather than equity- or share-holders. The U.S. government agrees to repay the face or principal or par amount of the security at maturity, plus coupon interest at semi-annual intervals. Treasury securities are often considered “riskless” investments given that the “full faith and credit” of the U.S. government backs these securities.

The security buyer can either hold the bond or note until maturity, at which time the face value becomes due; or, the bond or note may be sold in the secondary markets prior to maturity. In the latter case, the investor recovers the market value of the bond or note, which may be more or less than its face value, depending upon prevailing yields. In the meantime, the investor receives semi-annual coupon payments every six months.

<sup>1</sup> U.S. Treasury Note and Bond Futures are listed for trading on and subject to the rules and regulations of the Board of Trade of the City of Chicago, Inc. (CBOT). CBOT lists futures on Treasury securities covering a broad set of maturities, including the benchmark 10-Year Treasury Note futures. CME Group is the parent of four U.S. based futures exchanges that are registered under the Commodity Exchange Act (“CEA”) with the CFTC as designated contract markets (“DCMs”): Chicago Mercantile Exchange (“CME”), the Board of Trade of the City of Chicago, Inc. (“CBOT”), New York Mercantile Exchange, Inc. and Commodity Exchange, Inc. (collectively, the “CME Group Exchanges” or “Exchanges”).

## Treasury Futures Avg Daily Volume



E.g., you purchase \$1 million face value of the 2-1/4% note maturing in August 2027. This security pays half its stated coupon or 1-1/8% of par on each six-month anniversary of its issue. Thus, you receive \$22,500 (= 2-1/4% of \$1 million) annually, paid out in semi-annual installments of \$11,250 in February and August. Upon maturity in August 2027, the \$1 million face value is re-paid and the note expires.

## Price/Yield Relationship

A key factor governing the performance of bonds in the market is the relationship of yield and price movement. In general, as yields increase, bond prices will decline; as yields decline, prices rise. In a rising rate environment, bondholders will witness their principal value erode; in a decline rate environment, the market value of their bonds will increase.

**IF Yields Rise ↑ THEN Prices Fall ↓**

**IF Yields Fall ↓ THEN Prices Rise ↑**

This inverse relationship may be understood when one looks at the marketplace as a true auction. Assume an investor purchases a 10-year note with a 2% coupon when yields are at 2%. Thus, the investor pays 100% of the face or par value of the security. Subsequently, rates rise to 3%. The investor decides to sell the original note with the 2% yield, but no one will pay par as notes are now quoted at 3%. Now he must sell the note at a discount to par in order to move the note. I.e., rising rates are accompanied by declining prices.

Falling rates produce the reverse situation. If rates fall to 1%, our investment yields more than market rates. Now the seller can offer it at a premium to par. Thus, declining rates are accompanied by rising prices. Should you hold the note until maturity, you would receive the par or face value. In the meantime, of course, one receives semi-annual coupon payments.

## Quotation Practices

Unlike money market instruments (including bills and Eurodollars) that are quoted on a yield basis in the cash market; coupon-bearing securities are frequently quoted in percent of par to the nearest 1/32nd of 1% of par.

E.g., one may quote a bond or note at 97-18. This equates to a value of 97% of par plus 18/32nds. The decimal equivalent of this value is 97.5625. Thus, a one million-dollar face value security might be priced at \$975,625. If the price moves by 1/32nd from 97-18 to 97-19, this equates to a movement of \$312.50 (per million-dollar face value).

But often, these securities, particularly those of shorter maturities, are quoted in finer increments than 1/32nd. For example, one may quote the security to the nearest 1/64th. If the value of our bond or note in the example above were to increase from 97-18/32nds by 1/64th, it may be quoted at 97-18+. The trailing “+” may be interpreted as +1/64th.

Or, you may quote to the nearest 1/128th. If our note were to increase from 97-18/32nds by 1/128th, it might be quoted on a cash screen as 97-182. The trailing “2” may be read as +2/8ths of 1/32nd; or, 1/128th. If the security rallies from 97-18/32nds by 3/128ths, it may be quoted as 97-186. The trailing “6” may be read as +6/8ths of 1/32nd or 3/128ths. Sometimes, quotation systems use an alternate fractional reference. E.g., the value of 97-182 might be displayed as 97-18¼. Or a value of 97-18+ might be displayed as 97-18½. A value of 97-186 might be displayed as 97-18¾.

Futures quotation practices are similar but not entirely identical. A quote of 97-182 is the same no matter whether you are looking at a cash or a futures quote. It means 97% of par plus 18/32nds plus 1/128th.

## Quotation Practices

| Cash Price       | Means                  | Decimal Equivalent (% of Par) | Futures Quote |
|------------------|------------------------|-------------------------------|---------------|
| 97-18            | 97-18/32nds            | 97.5625000                    | 97-18         |
| 97-182 or 97-18¼ | 97-18/32nds + 1/128th  | 97.5703125                    | 97-182        |
| 97-18+ or 97-18½ | 97-18/32nds + 1/64th   | 97.5781250                    | 97-185        |
| 97-186 or 97-18¾ | 97-18/32nds + 3/128ths | 97.5859375                    | 97-187        |

But in the case of the cash markets, that trailing “2” means 2/8ths of 1/32nd = 1/128th. In the case of the futures markets that trailing “2” represents the truncated value of  $0.25 \times 1/32nd$  or 1/128th. A quote of 97-18+ in the cash markets is equivalent to 97-185 in the futures market. That trailing “5” represents  $0.5 \times 1/32nd$  or 1/64th. A quote of 97-186 in the cash markets is equivalent to 97-187 in the futures market. The trailing “7” represents the truncated value of  $0.75 \times 1/32nd = 3/128ths$ .

The normal commercial “round-lot” in the cash markets is \$1 million face value. Anything less might be considered an “odd-lot.” However, you can purchase Treasuries in units as small as \$1,000 face value. Of course, a dealer’s inclination to quote competitive prices may dissipate as size diminishes. Ultra Treasury bond, Treasury bond, Ultra 10-year, 10-year and 5-year Treasury note futures, however, are traded in units of \$100,000 face value. 3-year and 2-year Treasury note futures are traded in units of \$200,000 face value.

## Accrued Interest and Settlement Practices

In addition to paying the (negotiated) price of the coupon-bearing security, the buyer also typically compensates the seller for any interest accrued between the last semi-annual coupon payment date and the settlement date of the security.

E.g., it is October 10, 2017. You purchase \$1 million face value of the 2-1/4% Treasury security maturing in August 2027 (a ten-year note) for a price of 99-01 (\$990,312.50) to yield 2.36%, for settlement on the next day, October 11, 2017.

In addition to the price of the security, you must further compensate the seller for interest of \$3,485.05 accrued during the 57 days between the original issue date of August 15, 2017 and the settlement date of October 11, 2017.

This interest is calculated relative to the 57 days between issue date of August 15, 2017 and the next coupon payment date of February 15, 2018 or \$3,485.05 [= (57/184) x (\$22,500/2)]. The total purchase price is \$993,797.55.

|                         |              |
|-------------------------|--------------|
| <b>Price of Note</b>    | \$990,312.50 |
| <b>Accrued Interest</b> | \$3,485.05   |
| <b>Total</b>            | \$993,797.55 |

Typically, securities are transferred through the Fed wire system from the bank account of the seller to that of the buyer vs. cash payment. That transaction is concluded on the settlement date which may be different from the transaction date.

Unlike the futures market where trades are settled on the same day they are transacted, it is customary to settle a cash transaction on the business day subsequent to the actual transaction. Thus, if you purchase the security on a Thursday, you typically settle it on Friday. If purchased on a Friday, settlement will generally be concluded on the following Monday.

Sometimes, however, a “skip date” settlement is specified. E.g., one may purchase a security on Monday for skip date settlement on Wednesday. Or, “skip-skip date” settlement on Thursday; “skip-skip-skip date” settlement on the Friday, etc. Skip or forward date settlements may be useful in order to match Treasury transaction payments with one’s anticipated future cash flows at current market prices. Theoretically, there is no effective limitation on the number of days over which one may defer settlement. Thus, these cash securities may effectively be traded as forward contracts.

## Treasury Auction Cycle

Treasury securities are auctioned on a regular basis by the U.S. Treasury which accepts bids on a yield basis from security dealers. A certain amount of each auction is set aside, to be placed on a non-competitive basis at the average yield filled.

Prior to the actual issuance of specific Treasuries, they may be bought or sold on a “WI” or “When Issued” basis. When traded on a WI basis, bids and offers are quoted as a yield rather than as a price in anticipation of the establishment of the coupon subsequent to the original auction.

## U.S. Treasury Auction Schedule

### Fixed-Principal, Fixed-Coupon Notes and Bonds

|                       | <b>Maturity</b>       | <b>Auctioned</b>  |
|-----------------------|-----------------------|---|
| <b>Treasury Notes</b> | 2-, 3-, 5- and 7-Year | Monthly   |
|                       | 10-Year               | February, May, August & November with re-openings in other 8 months |
| <b>Treasury Bonds</b> | 30-Year               | February, May, August & November with re-openings in other 8 months |

Security dealers purchase these recently auctioned securities and subsequently market them to their customers including pension funds, insurance companies, banks, corporations and retail investors. The most recently issued securities of a particular maturity are referred to as “on-the-run” securities. On-the-runs are typically the most liquid and actively traded of Treasury securities and, therefore, are often referenced as pricing benchmarks. Less recently issued securities are known as “off-the-run” securities and tend to be less liquid.

The Treasury currently issues 4-week, 13-week, 26-week and 52-week bills; 2-year, 3-year, 5-year, 7-year and 10-year notes; 2-year Floating Rate Notes (FRNs); and, 30-year bonds on a regular schedule. In the past, the Treasury had also issued securities with a 4-year and 20-year maturity. Further, the Treasury may issue very short term cash management bills along with Treasury Inflation Protected Securities or “TIPS.”

## The “Run”

If you were to ask a cash dealer for a quotation of “the run,” he would quote yields associated with the on-the-run securities from the current on-the-run Treasury bills, through notes, all the way to the 30-year bond, sometimes referred to as the “long-bond” because it is the longest maturity Treasury available.

### Quoting ‘the Run’ (As of October 10, 2017)

|                   | <b>Coupon</b> | <b>Maturity</b> | <b>Price</b> | <b>Yield</b> |
|-------------------|---------------|-----------------|--------------|--------------|
| <b>2-Yr Note</b>  | 1-3/8%        | 09/30/19        | 99-237       | 1.506%       |
| <b>3-Yr Note</b>  | 1-3/8%        | 09/15/20        | 99-08+       | 1.630%       |
| <b>5-Yr Note</b>  | 1-7/8%        | 09/30/22        | 99-21+       | 1.943%       |
| <b>7-Yr Note</b>  | 2-1/8%        | 09/30/24        | 99-19+       | 2.190%       |
| <b>10-Yr Note</b> | 2-1/4%        | 08/15/27        | 99-01        | 2.360%       |
| <b>30-Yr Bond</b> | 2-3/4%        | 08/15/47        | 97-122       | 2.881%       |

The most recently issued security of any tenor may be referred to as the “new” security. Thus, the second most recently issued security of a particular original tenor may be referred to as the “old” security, the third most recently issued security is the “old-old” security, the fourth most recently issued security is the “triple old” security.

As of October 10, 2017, the most recently issued 10-year note was identified as the 2-1/4% note maturing in August 2027; the old note was the 2-3/8% note of May 2027; the old-old note was the 2-1/4% of February 2027; the triple old note was the 2% of November 2026.

Beyond that, one is expected to identify the security of interest by coupon and maturity. For example, the “2s of ‘26” refers to the note with a coupon of 2% maturing on November 15, 2026. As of October 10, 2017, there were not any “WI” or “when issued” 10-year notes.

### 10-Year Treasury Notes (As of October 10, 2017)

|                   | Coupon     | Maturity   | Price   | Yield |
|-------------------|------------|------------|---------|-------|
| <b>On-the-Run</b> | 2-1/4%     | 8/15/2027  | 99-01   | 2.36% |
| <b>Old Note</b>   | 2-3/8%     | 5/15/2027  | 100-05+ | 2.35% |
| <b>Old-Old</b>    | 2-1/4%     | 2/15/2027  | 99-05+  | 2.35% |
| <b>Triple Old</b> | 2%         | 11/15/2026 | 97-07+  | 2.34% |
|                   | 1-1/2%     | 8/15/2026  | 93-14   | 2.32% |
|                   | 1-5/8%     | 5/15/2026  | 94-21+  | 2.31% |
|                   | 1-5/8%     | 2/15/2026  | 94-29   | 2.30% |
|                   | 2-1/4%     | 11/15/2025 | 99-25   | 2.28% |
|                   | 2%         | 8/15/2025  | 98-03   | 2.26% |
|                   | 2-1/8%     | 5/15/2025  | 99-05+  | 2.24% |
|                   | 2%         | 2/15/2025  | 98-14+  | 2.23% |
|                   | 2-1/4%     | 11/15/2024 | 100-09+ | 2.20% |
| 2-3/8%            | 08/15/2024 | 101-07+    | 2.18%   |       |

### The Roll and Liquidity

Clearly, traders who frequently buy and sell are interested in maintaining positions in the most liquid securities possible. As such, they tend to prefer on-the-run (OTR) as opposed to off-the-run securities.

It is intuitive that on-the-runs will offer superior liquidity when one considers the “life-cycle” of Treasury securities. Treasuries are auctioned, largely to broker-dealers, who subsequently attempt to place the securities with their customers. Often these securities are purchased by investors who may hold the security until maturity. At some point, securities are “put-away” in an investment portfolio until their maturity. Or, they may become the subjects of

a strip transaction per the STRIPS (Separate Trading of Registered Interest and Principal on Securities) program.

As these securities find a home, supplies may become scarce. As a result, bid/offer spreads may inflate and the security becomes somewhat illiquid. Liquidity is a valuable commodity to many. Thus, you may notice that the price of on-the-runs may be bid up, resulting in reduced yields, relative to other similar maturity securities. This tends to be most noticeable with respect to the 30-year bond.

Traders may be interested in conducting a “roll” transaction where one sells the old security in favor of the new security, in order to maintain a position in the on-the-run and most liquid security. Thus, dealers will quote a bid/offer spread in the roll, offering the opportunity to sell the old note/buy the new note; or, buy the old note/sell the new note, in a single transaction.

The “old note” in our table above was quoted at a yield of 2.35% while the “new note” was seen at 2.36%. In this case, the roll is quoted at approximately negative 1 basis point ( $-0.01\% = 2.35\% - 2.36\%$ ).

### Repo Financing

Leverage is a familiar concept to futures traders. Just as one may margin a futures position and thereby effectively extend one’s capital, the Treasury markets likewise permit traders to utilize “repo” financing agreements to leverage Treasury holdings.

A repurchase agreement, repo or simply RP represents a method by which one may borrow funds, typically on a very short-term basis, collateralized by Treasury securities. In a repo agreement, the lender will wire transfer same-day funds to the borrower; the borrower wire transfers the Treasury security to the lender with the provision that the transactions are reversed at term with the borrower wiring back the original principal plus interest.

The borrower is said to have executed a repurchase agreement; the lender is said to have executed a reverse repurchase agreement. Many banks and security dealers will offer this service, once the customer applies and passes a requisite credit check.

The key to the transaction, however, is the safety provided the lender by virtue of the receipt of the (highly-marketable) Treasury security. These repo transactions are typically done on an overnight basis but may be negotiated for terms such as one-week, two-weeks, or one month. A third party custodian is frequently used to add an additional layer of safety between the lender and borrower, i.e., a tri-party repo agreement. Overnight repo rates are typically quite low, in the vicinity of the effective federal funds rate.

Any Treasury security may be considered “good” or “general” collateral. Sometimes when particular Treasuries are in short supply, dealers will announce that the security is “on special” and offer below-market financing rates in an effort to attract borrowers.

## Treasury Futures Delivery Practices

While some traders refer to original or “classic” Treasury bond futures as “30-year bond futures,” that reference is actually quite misleading. Treasury bond futures permit the delivery in satisfaction of a maturing contract of any U.S. Treasury security provided it matures within a range of 15 to less than 25 years from the date of delivery. That delivery window once reduced from 15 to 30 years and, thus, the characterization of the Treasury bond contract as a “30-year bond futures” was apt.

Note that the Ultra T-bond futures contract calls for the delivery of any bond that does not mature for a period of at least 25 years from the date of delivery. Subsequent to the development of the Ultra bond contract, the delivery window of the original T-bond futures contract was amended from 15-30 years to 15-25 years. As such, the Ultra T-bond futures contract currently is most aptly referred to as the 30-year bond contract while the original bond futures contract, as amended, is referred to as the “classic” bond futures contract.

Because of the rather broadly defined delivery specifications, a significant number of securities, ranging widely in terms of coupon and maturity, may be eligible for delivery. This applies with equal effect to 2-, 3-, 5-, 10-, and Ultra 10-year Treasury note futures; as well as the classic and Ultra T-bond futures contracts. Table 2 included below provides a complete description of the contract specifications of CME Group Treasury futures products.

## Conversion Factor Invoicing System

Securities with varying characteristics, such as coupon and maturity, will of course be more or less valued by the investment community. High-coupon securities, for example, will naturally command a greater price than comparable low-coupon securities.

These differences must be reflected in the futures contract. In particular, when a short makes delivery of securities in satisfaction of a maturing futures contract, the long will pay a specified invoice price to the short.

As discussed above, the futures contract permits the delivery of a wide range of securities at the discretion of the short. That invoice value must be adjusted to reflect the specific pricing characteristics of the security that is tendered.

Accordingly, Treasury futures utilize a “conversion factor” invoicing system to reflect the value of the security that is tendered by reference to the 6% futures contract standard. The “Principal Invoice Amount” paid from long to short upon delivery may be identified as the Futures Settlement Price multiplied by the Conversion Factor (CF) multiplied by \$1,000.

That \$1,000 constant reflects the \$100,000 face value futures contract size associated with most T-note and T-bond futures. Note that the 2-year T-note contract is based on a \$200,000 face value amount. Thus, this constant must be reset at \$2,000 for 2-year Treasury futures.

$$\begin{aligned} \text{Principal Invoice Price} \\ &= \text{Futures Settlement} \times \text{Conversion Factor (CF)} \\ &\quad \times \$1,000 \end{aligned}$$

Any interest accrued since the last semi-annual interest payment date is added to the principal invoice amount to equal the “total invoice amount.”

$$\begin{aligned} \text{Total Invoice Amount} \\ &= \text{Principal Invoice Amount} \\ &\quad + \text{Accrued Interest} \end{aligned}$$

A conversion factor may be thought of as the clean price of \$1 face value of the delivered security at a yield to maturity of 6%. Clearly, high-coupon securities will tend to have high CFs while low-coupon securities will tend to have low CFs. In particular, bonds or notes with coupons less than the 6% contract standard will have CFs that are less than 1.0; bonds or notes with coupons greater than 6% have CFs greater than 1.0.

E.g., the conversion factor for delivery of the 2-3/8% T-note of Aug-24 vs. December 2017 10-year T-note futures is 0.8072. This suggests that a 2-3/8% security is approximately valued at 81% as much as a 6% security. Assuming a futures price of 125-08+32nds (or 125.265625 expressed in decimal format), the principal invoice amount may be calculated as follows.

**Principal Invoice Price**

$$= 125.265625 \times 0.8072 \times \$1,000$$

$$= \$101,114.41$$

E.g., the conversion factor for delivery of the 1-7/8% T-note of Aug-24 vs. December 10-year T-note futures is 0.7807. This suggests that a 1-7/8% security is approximately valued at 78% as much as a 6% security. Assuming a futures price of 125-08+32nds (or 125.265625), the principal invoice amount may be calculated as follows.

**Principal Invoice Price**

$$= 125.265625 \times 0.7807 \times \$1,000$$

$$= \$97,794.87$$

In order to arrive at the total invoice amount, one must of course further add any accrued interest since the last semi annual interest payment date to the principal invoice amount.

**Cheapest-to-Deliver**

The intent of the conversion factor invoicing system is to render equally economic the delivery of any eligible-for-delivery securities. Theoretically, the short who has the option of delivering any eligible security should be indifferent as to his selection.

However, the CF system is imperfect in practice as we find that a particular security will tend to emerge as “cheapest-to-deliver” (CTD) after studying the relationship between cash security prices and principal invoice amounts.

E.g., on October 10, 2017, one might have been able to purchase the 2-3/8% of 8/24 at 101-07+ (\$101,234.38 per \$100,000 face value unit). The 1-7/8% of 8/24 was valued at 98-01+ (\$98,031.25 per \$100,000 face value unit). Compare these cash values to the principal invoice amounts as follows.

|                           | 2-3/8%-8/24       | 1-7/8%-8/24       |
|---------------------------|-------------------|-------------------|
| <b>Futures Price</b>      | 125-085           | 125-085           |
| <b>x CF</b>               | 0.8072            | 0.7807            |
| <b>x \$1,000</b>          | \$1,000           | \$1,000           |
| <b>Principal Invoice</b>  | \$101,114.41      | \$97,794.87       |
| <b>Cash Price</b>         | (\$101,234.38)    | (\$98,031.25)     |
| <b>Delivery Gain/Loss</b> | <b>(\$119.97)</b> | <b>(\$236.38)</b> |

Our analysis suggests that a loss of \$119.97 may be associated with the delivery of the 2-3/8% of 8/24 while an even larger loss of \$236.38 might be associated with the delivery of the 1-7/8% of 8/24. Thus, we might conclude that the 2-3/8% of 8/24 note is cheaper or more economic to deliver than the 1-7/8% of 8/24.

**The Basis**

Typically, we expect to find a single security, or perhaps a handful of similar securities, will emerge as CTD. This identification has important implications for basis traders who arbitrage cash and futures markets. A basis trader will seek out arbitrage opportunities or situations where they might be able to capitalize on relatively small pricing discrepancies between cash securities and Treasury futures by buying “cheap” and selling “rich” items.

Arbitrageurs will track these relationships by studying the “basis.” The basis describes the relationship between cash and futures prices and may be defined as the cash price less the “adjusted futures price” or the futures price multiplied by the conversion factor.

Thus, the basis is analogous to the gain or loss that might be realized upon delivery. Unlike that gain or loss, however, the basis is typically expressed in terms of 32nds. E.g., 1-1/4 points might be shown as 40/32nds. It is also “inverted” in the sense that we are comparing cash less adjusted futures prices - rather than futures invoice price less cash prices.

**Basis = Cash Price-Adjusted Futures Price**

**Adjusted Futures Price = Futures Price x Conversion Factor**

E.g., a comparison of cash and adjusted futures prices provides us with a quote for the basis associated with the 2-3/8%-8/24 and 1-7/8%-8/24 Treasury securities.

|                  | 2-3/8%-24    | 1-7/8%-24    |
|------------------|--------------|--------------|
| Cash Price       | 101-07+      | 98-01+       |
| Futures Price    | 125-085      | 125-085      |
| x CF             | 0.8072       | 0.7807       |
| Adjusted Futures | (101-04)     | (97-255)     |
| Basis (32nds)    | <b>3.339</b> | <b>8.064</b> |

The basis of 3.339/32nds associated with the 2-3/8%-8/24 corresponds to a loss on delivery of \$119.97 as shown above. Similarly, the basis of 8.064/32nds associated with the 1-7/8%-8/24 corresponds to a loss on delivery of \$236.38.

As suggested above, and as a general rule, the security with the lowest basis (and highest implied repo rate), i.e., the largest gain or smallest loss on delivery, may be considered CTD. Clearly, the 2-3/8%-8/24 is cheaper-to-deliver than the 1-7/8%-8/24.

Table 3 included at the end of this document depicts the basis and implied repo rates for all eligible-for-delivery securities vs. the December 2017 10-year T-note futures contract as of October 10, 2017. Referring to Table 3, one may confirm that the 2-3/8%-8/24 exhibited the lowest basis and, therefore, may be considered the CTD security.

Note, however, that there are quite a few securities, with similar coupons and maturities, which are near CTD. In fact, the entire universe of eligible-for-delivery securities features reasonably similar coupons and maturities.

It is important to identify the CTD security to the extent that Treasury futures will tend to price or track or correlate most closely with the CTD. This has interesting implications from the standpoint of a “basis trader” or a hedger as discussed in more detail below.

Suffice it to say at this point that basis trading is a frequent practice in the Treasury futures markets. Certain terminology has been developed to identify basis positions. One may “buy the basis” by buying cash securities and selling futures. One may “sell the basis” by selling cash securities and buying futures.

Basis transactions are typically transacted in a ratio that reflects the conversion factor of the security involved in the trade.

**“Buy the Basis” = Buy cash securities & sell futures**

**“Sell the Basis” = Sell cash securities & buy futures**

E.g., if one were to buy the basis by buying \$10 million face value of the 2-3/8%-8/24 note, one might sell 81 December 2017 futures by reference to the conversion factor of 0.8072.

E.g., if one were to sell the basis by selling \$10 million face value of the 1-7/8%-8/24 note, one might buy 78 December futures by reference to the conversion factor of 0.7807.

By transacting the basis in a ratio identified by reference to the CF, one may roughly balance the movement or volatility on both legs of the spread. This is intuitive to the extent that the conversion factor generally reflects the value of the cash position relative to that of the futures contract. If the CF reflects relative value then presumably it will reflect relative volatility or price movement as well.

## Why Is One Issue CTD?

If the conversion factor invoicing system performed flawlessly, all eligible-for-delivery securities would have a similar basis and be equally economic to deliver. As suggested above, however, a single security or several similar securities tend to emerge as CTD.

The CF invoicing system is imperfect because it is implicitly based on the assumption that - (1) all eligible-for-delivery securities have the same yield; and (2) that yield is 6%. But there are any number of “cash market biases” that impact upon the yield of a Treasury security.

Further mathematical biases in the conversion factor calculation will tilt the field towards securities of particular coupons and maturities when yields are greater than or less than the 6% contract standard. Hence, we may further speak of “conversion factor biases.”

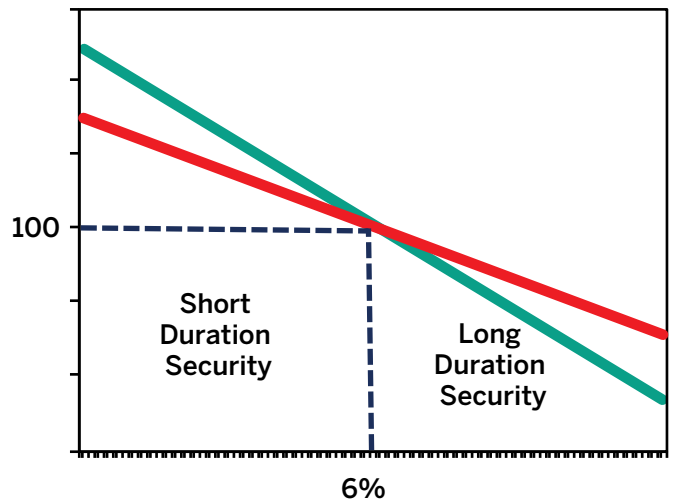
## Conversion Factor Effects

Perhaps more important than these cash market factors, there are observable biases associated with the mathematics of the conversion factor system or conversion factor biases.

Long duration, i.e., low-coupon, long-maturity securities, will become CTD when yields are significantly greater than the 6% contract standard. When yields fall below the 6% contract standard, these factors will bias towards the delivery of short-duration, i.e., high-coupon, short-maturity securities.

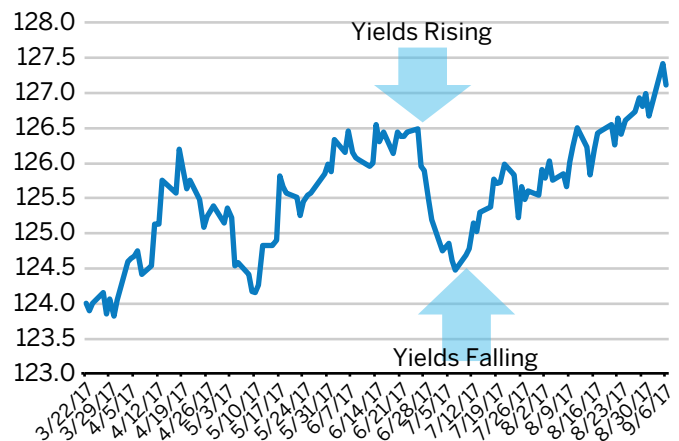
|                            |   |
|----------------------------|---|
| <p>If yields &gt; 6% →</p> | <p><b>Bias to long duration</b><br/>(i.e., low-coupon,<br/>long-maturity) securities</p>    |
| <p>If yields &lt; 6% →</p> | <p><b>Bias to short duration</b><br/>(i.e., high-coupon,<br/>short-maturity) securities</p> |

## CTD Driven by Yields



Consider the period between March and September 2017 as depicted in our graphic. During this period, the price of the December 2017 Ten-year T-note futures experienced a price decline in late June 2017 from approximately 126.5% to 124.5% of par. Subsequently, the market has trended higher to a new high of nearly 127.5% of par.

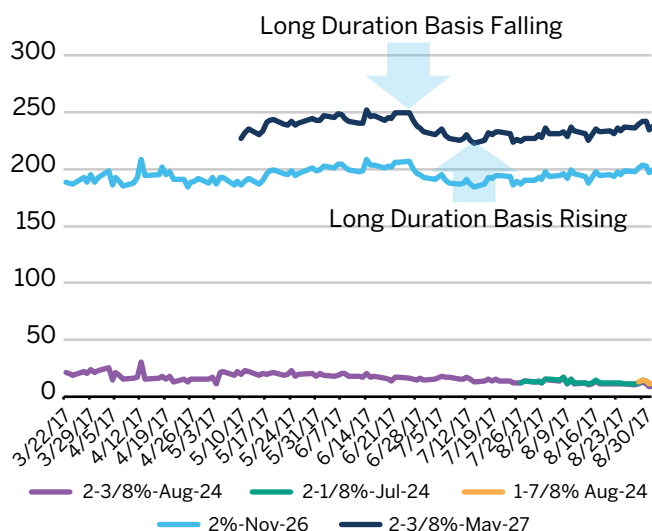
## Dec-17 10-Yr Note Futures



In other words, prices dropped briefly as yields spiked, only to be followed by an extended period of prices rising as yields fell. During the entirety of this period, prices were well above par while yields were well below the 6% futures contract standard. Still, conversion factor biases were diminished or weakened as prices strengthened only to decline once again as the market broke.

The impact of these strengthening and subsequently weakening conversion factor biases may be observed by examining the basis for several eligible-for-delivery securities. Actually, the simple and gradual convergence of cash and futures prices may be the feature that is most apparent from an examination of this graphic.

### Dec-17 10-Yr Basis



As prices advanced and yields fell since late June, notice that the basis was buoyed upwards to the extent that its price rose faster than futures price which traced a shorter duration CTD. Again, as yields fall below or further below the 6% futures contract standard, long duration securities tend to become less economic to deliver.

| Yields Rising above 6%   | Yields Falling Under 6%   |
|--|---|
| Sell long duration basis, i.e., sell long duration securities & buy futures  | Buy long duration basis, i.e., buy long duration securities & sell futures    |
| Buy short duration basis, i.e., buy short duration securities & sell futures | Sell short duration basis, i.e., sell short duration securities & buy futures |

As prices declined and yields rose in late June, the basis for long duration securities such as the 2%-26 or the 2-3/8%-27 tended to decline more sharply than the basis for short duration securities such as the CTD 2-3/8%-8/24. This is consistent with our observation above that, as yields rise, long duration securities tend to become more economic to deliver.

It is clear that the performance of the basis is strongly driven by directional price movement in the Treasury markets. Thus, “buying the basis” or “selling the basis” may be motivated by expectations regarding rising or falling yields. The key is to get a sense of market direction and then identify the long or short duration securities whose basis values will be impacted by any sizable price (or yield) movement.

### Implied Repo Rate

We often suggest that the security with the lowest basis is cheapest-to-deliver. But to be more precise, we may point out that the structure of coupon receipts and reinvestment of such coupon income plays some (generally small) part in establishing a particular security as cheapest-to-deliver as well. Hence, traders often calculate the “implied repo rate” (IRR) associated with eligible for delivery securities to account for such factors.

The IRR is calculated as the annualized rate of return associated with the purchase of a security, sale of futures and delivery of the same in satisfaction of the maturing futures contract. This calculation takes into account all the cash flows associated with the security. The assumption that the basis for any particular security may completely converge to zero is implicit in the IRR calculation.

E.g., if one were to buy the 2-3/8%-8/24 basis by buying the cash securities, selling futures in a ratio dictated by the conversion factor and making delivery, or at least witnessing full cash-futures convergence, one would lock-in a return of 1.42%.

E.g., if one were to buy the 1-7/8%-8/24 basis by buying cash securities and selling futures in a ratio indicated by reference to the conversion factor and making delivery, or at least witnessing full cash-futures convergence, one would lock-in a rate of return of 0.32%.

Clearly, it would be preferable to lock-in a return of 1.42% rather than a return of 0.32%. Thus, the 2-3/8%-8/24 is cheaper to deliver relative to the 1-7/8%-8/24. In fact, if we scan the IRRs associated with all securities eligible to be delivered into the December 2017 contract in Table 3 below, we find that the IRR of 1.42% associated with the 2-3/8%-8/24 is superior to all other IRRs.

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Thus, the 2-3/8%-8/24 Treasury security is associated with the lowest basis and the highest IRR as of October 10, 2017. As a general rule, the security with the lowest basis will likewise exhibit the highest implied repo rate. It is possible that a security with the lowest basis may not quite have the highest IRR because of cash flow considerations. But this statement is generally true. In any event, this observation confirms the CTD status of the 2-3/8%-8/24 as of October 10, 2017.

By buying the basis of a Treasury security, or buying cash and selling futures, one becomes obligated to make delivery of the Treasury in satisfaction of the maturing futures contract.<sup>2</sup> Thus, buying the basis of the cheapest-to-deliver 2-3/8%- 8/24 vs. a futures contract that matures two or three months hence, may be considered analogous to other short-term investment alternatives.

E.g., we might compare the IRR = 1.42% associated with the CTD security to the prevailing 13-week T-bill yield of 1.04%; or to the effective Fed Funds rate of 1.15%; or, to a 3-month ICE LIBOR rate at 1.34%.

In this example, the IRR associated with the CTD security was essentially equivalent to other short-term investment opportunities. As a general rule, however, the IRR even for the CTD security tends to run at a level that is a bit inferior to the returns associated with comparable short-term investment alternatives. The IRRs associated with all other non CTD securities are even lower.

This begs the question – why would anyone ever want to buy the basis if the returns do not appear to be competitive? The answer lies in the fact that the basis conveys other opportunities apart simply from the opportunity to use the futures contract as a delivery conveyance.

Consider any discrepancy with respect to the CTD to represent a risk premium of sorts. If one buys the CTD security and sells futures with the intention of making delivery, the worst case scenario has the basis converging fully to zero and the hedger essentially locking in a return equal to the IRR, in this case 1.42%.

But if market conditions should change such that another security becomes CTD, this implies that the basis may advance, or at least fail to completely converge to zero. As a result, the trader may realize a rate of return that is in fact greater than the currently calculated IRR.

<sup>2</sup> One may, of course, opt to offset the short futures contract prior to the delivery period and effectively abrogate such obligation.

## Basis Optionality

In other words, there is a certain degree of “optionality” associated with the purchase or sale of the basis. Buying the basis is analogous to buying an option which, of course, implies limited risk. Buying the basis implies limited risk to the extent that, even under the worst of circumstances, you make delivery of the security which is effectively equivalent to the possibility that the basis fully converges to zero.

But “crossovers” or “switch” may occur such that the basis converges at a slower rate than otherwise anticipated or actually advances. As a result, this short-term investment may generate a return which is (at least theoretically) unbounded on the upside. Limited risk accompanied by unbounded upside potential is reminiscent of the risk/reward profile of a long option position, thus the analogy between a long basis position and a long option.

The best one may hope by selling the basis, or selling securities and buying futures with the possibility of effectively replacing the sold security by standing long in the delivery process, is that the basis fully converges to zero. This implies limited profit potential.

But in the event of significant changes in market conditions, the basis may advance sharply, exposing the seller of the basis to (theoretically) unbounded risks. Limited profit potential accompanied by unbounded risk is reminiscent of the risk/reward profile of a short option position, thus the analogy between a short basis position and a short option.

As discussed above, the basis even for the CTD security tends to be in excess of cost of carry considerations. This is manifest in the fact that the IRR even for the CTD is typically a bit below prevailing short-term rates. This premium in the basis essentially reflects the uncertainties associated with which security may become CTD in the future.

Thus, the basis performs much akin to an option. Like any other option, the basis will be affected by considerations including term, volatility and strike price. The relevant term in this case is the term remaining until the presumed delivery date vs. the futures contract. Market volatility affects the probability that a crossover may occur. Rather than speak of a strike or exercise price, it is more appropriate to assess the market’s proximity to a “crossover point” or a price/yield at which one might expect an alternate security to become CTD.

Consider the purchase or sale of the CTD basis. The degree to which this basis performs like a call or a put option is contingent upon the relationship between market prices and the 6% futures contract standard.

If yields are below the 6% futures contract standard, the CTD basis may be expected to advance if prices decline (rates rise) towards 6%; or, decline if prices advance (rates fall). Thus, buying the CTD basis when rates are below 6% is akin to the purchase of a put option. Conversely, the sale of the CTD basis when rates are less than 6% is akin to the sale of a put option where the value of transaction is capped if prices should advance while losses may be unbounded if prices should decline.

If yields are above the 6% futures contract standard, the CTD basis may be expected to advance if prices rise (rates fall) towards 6%; or, decline if prices fall (rates rise). Thus, buying the CTD basis when rates are above 6% is akin to the purchase of a call option. Conversely, the sale of the CTD basis when rates are above 6% is akin to the sale of a call option where the value of transaction is capped if prices should decline while losses may be unbounded if prices should advance.

Finally, if rates are close to the 6% futures contract standard, the basis for what is currently CTD may be dictated by considerations apart from conversion factor biases.

Thus, there may be significant crossovers regardless of whether rates rise or fall. Buying the CTD basis under these considerations may be considered akin to the purchase of an option straddle (i.e., the simultaneous purchase of call and put options).

Under these circumstances the basis buyer may be indifferent between advancing or declining prices but has an interest in seeing prices move significantly in either direction. Selling the CTD basis when rates are near the 6% contract standard is akin to selling a straddle (i.e., the simultaneous sale of both call and put options). The basis is sold under these circumstances because the trader anticipates an essentially neutral market.

|                       | Buy CTD Basis   | Sell CTD Basis   |
|-----------------------|-----------------|------------------|
| <b>Yields &lt; 6%</b> | Buy Put Option  | Sell Put Option  |
| <b>Yields = 6%</b>    | Buy Straddle    | Sell Straddle    |
| <b>Yields &gt; 6%</b> | Buy Call Option | Sell Call Option |

Of course, the basis premium over carry should accrue to the short basis trader under circumstances of continued price stability. But the short basis trader is exposed to the risk of dramatic price movements in either direction.

As of October 10, 2017, the IRR of the CTD 2-3/8%-8/24 security at 1.42% fell squarely within the range of other short-term investment alternatives. This suggests negligible optionality, i.e., the probability of a crossover or switch is negligible. This is driven by the fact that yields are well below the 6% futures contract standard. Further, the duration of the 2-3/8%-8/24, with its high coupon and short maturity, was the shortest relative to other eligible for delivery securities. Thus, the market assessed a negligible probability that this security would not remain CTD by the time we enter the December 2017 delivery period.

## Measuring Risk

There are a couple of popular ways to measure the risks associated with coupon-bearing (and money-market) instruments including basis point value (BPV) and duration.

## Basis Point Value (BPV)

BPV represents the absolute price change of a security given a one basis point (0.01%) change in yield. These figures may be referenced using any number of commercially available quotation services or software packages. BPV is normally quoted in dollars based on a \$1 million (round-lot) unit of cash securities. The following table depicts the BPVs of various on-the-run Treasuries as of October 30, 2017.

## Measuring Volatility (As of October 30, 2017)

|            | Coupon | Maturity | Duration (Yrs) | BPV (per mil) |
|------------|--------|----------|----------------|---------------|
| 2-Yr Note  | 1-1/2% | 10/31/19 | 1.978          | \$196         |
| 3-Yr Note  | 1-5/8% | 10/15/20 | 2.896          | \$287         |
| 5-Yr Note  | 2%     | 10/31/22 | 4.783          | \$473         |
| 7-Yr Note  | 2-1/4% | 10/31/24 | 6.516          | \$645         |
| 10-Yr Note | 2-1/4% | 08/15/27 | 8.794          | \$863         |
| 30-Yr Bond | 2-3/4% | 08/15/47 | 20.241         | \$1,946       |

*E.g., this suggests that if the yield on the 30-year bond were to rise by a single basis point (0.01%), the price should decline by some \$1,946 per \$1 million face value unit.*

## Duration

If BPV measures the absolute change in the value of a security given a yield fluctuation; duration may be thought of as a measure of relative or percentage change. The duration (typically quoted in years) measures the expected percentage change in the value of a security given a one-hundred basis point (1%) change in yield.

Duration is calculated as the average weighted maturity of all the cash flows associated with the bond, i.e., repayment of "corpus" or face value at maturity plus coupon payments, all discounted to their present value.

E.g., the 30-year bond is associated with duration of 20.2 years. This implies that if its yield advances by 100 basis points (1.00%), we expect a 20.2% decline in the value of the bond.

In years past, it was commonplace to evaluate the volatility of coupon-bearing securities simply by reference to maturity. But this is quite misleading. If one simply examines the maturities of the current 2-year note and 10-year note, one might conclude that the 10-year is 5 times as volatile as the 2-year.

But by examining durations, we reach a far different conclusion. The 10-year note (duration of 8.794 years) is only about 4-½ times as volatile as the 2-year note (duration of 1.978 years). The availability of cheap computing power has made duration analysis as easy as it is illuminating.

## Risk Management

Treasury futures are intended to provide risk averse fixed income investors with the opportunity to hedge or manage the risks inherent in their investment activities. Effective use of these contracts, however, requires a certain grounding in hedge techniques.

Most pointedly, one may attempt to assess the relative volatility of the cash item to be hedged relative to the futures contract price. This relationship is often identified as the futures "Hedge Ratio" (HR). Hedge ratios reflect the expected relative movement of cash and futures and provide risk managers with an indication as to how many futures to use to offset a cash exposure.

## CF Weighted Hedge

Treasury futures contract specifications conveniently provide a facile means by which to assess the relative risks associated with cash and futures. As discussed above, the conversion factor (CF) represents the price of a particular bond as if it were to yield 6%. Thus, the CF reflects the relative value and, by implication, the relative volatility between cash and futures prices. Most basis trades are in fact concluded in a ratio identified by reference to the CF.

E.g., if one held \$10 million face value of the 2-3/8%-8/24 note, one might sell 81 December 2017 futures by reference to the conversion factor of 0.8072 to execute a hedge.

E.g., if one held \$10 million face value of the 1-7/8%-8/24 note, one might sell 78 December 2017 futures by reference to the conversion factor of 0.7807 to execute a hedge.

A conversion factor weighted hedge is likely to be quite effective if you are hedging the cheapest-to-deliver security. Treasury futures will tend to price or track or correlate most closely with the CTD security.

But other securities with different coupons and maturities may react to changing market conditions differently. Thus, one might question if you can or should do better than a CF weighted hedge?

## BPV Weighted Hedge

In order to understand the most effective techniques with which to apply a hedge, consider the fundamental objective associated with a hedge. An “ideal” hedge is intended to balance any loss (profit) in the cash markets with an equal and opposite profit (loss) in futures.

Our goal, therefore, is to find a hedge ratio (HR) that allows one to balance the change in the value of the cash instrument to be hedged ( $\Delta_{\text{hedge}}$ ) with any change in the value of the futures contract ( $\Delta_{\text{futures}}$ ). Note that we use the Greek letter delta or  $\Delta$  to denote the abstract concept of change in value.

$$\Delta_{\text{hedge}} = HR \times \Delta_{\text{futures}}$$

We solve for the hedge ratio (HR) as follows.

$$HR = \Delta_{\text{hedge}} \div \Delta_{\text{futures}}$$

Because we have not defined what we mean by “change in value,” the equation above is of an abstract nature and cannot be directly applied. Thus, let’s backtrack to discuss the relationship between Treasury futures and cash prices.

Per our discussion above, principal invoice amount paid from long to short upon deliver will be equal to the futures price multiplied by the conversion factor of the cash security being delivered. Rational shorts will, of course, elect to tender the cheapest-to-deliver security. Thus, we might designate the futures price and the conversion factor of the cheapest-to-deliver as  $P_{\text{futures}}$  and  $CF_{\text{ctd}}$ , respectively.

$$\text{Principal Invoice Price} = P_{\text{futures}} \times CF_{\text{ctd}} \times \$1,000$$

Because the basis of the CTD is generally closest to zero, relative to all other eligible securities, we might assume that the futures price level and, by implication, any changes in the futures price level ( $\Delta_{\text{futures}}$ ) will be a reflection of any changes in the value of the CTD ( $\Delta_{\text{ctd}}$ ) adjusted by its conversion factor ( $CF_{\text{ctd}}$ ) as follows.

$$\Delta_{\text{futures}} = \frac{\Delta_{\text{ctd}}}{CF_{\text{ctd}}}$$

Substituting this quantity into our equation specified above, we arrive at the following formula.

$$HR = \Delta_{\text{hedge}} \div \left( \frac{\Delta_{\text{ctd}}}{CF_{\text{ctd}}} \right)$$

We might further rearrange the equation as follows.

$$HR = CF_{\text{ctd}} \times \left( \frac{\Delta_{\text{hedge}}}{\Delta_{\text{ctd}}} \right)$$

Unfortunately, this concept of “change in value” remains abstract. Let us “operationalize” the concept by substituting the basis point value of the hedged security ( $BPV_{\text{hedge}}$ ) and the basis point value of the cheapest-to-deliver ( $BPV_{\text{ctd}}$ ) for that abstract concept.

Recall from our discussion above that a basis point value represents the expected change in the value of a security, expressed in dollars per \$1 million face value, given a one basis point (0.01%) change in yield. Thus, we identify the basis point value hedge ratio (or “BPV HR”) as follows.

$$BPV \text{ HR} = CF_{\text{ctd}} \times \left( \frac{BPV_{\text{hedge}}}{BPV_{\text{ctd}}} \right)$$

Our analysis implicitly assumes that any changes in the yield of the hedged security and that of the cheapest-to-deliver security will be identical. I.e., that we will experience “parallel” shifts in the yield curve. This analysis further presumes that you are able to identify the cheapest-to-deliver security and that it will remain cheapest-to-deliver. The latter assumption is, of course, questionable in a dynamic market.

E.g., let us find the basis point value hedge ratio (HR) required to hedge \$10 million face value of the 2-3/8%-5/27 note security. This security carried a  $BPV = \$8,558$  per \$10 million. The CTD security was the 2-3/8%-8/24 with a  $BPV = \$63.78$  per \$100,000 face value and a conversion factor of 0.8072 vs. December 2017 Ten-year T-note futures. The hedge ratio may be identified as 108 contracts per \$10 million face value of the 2-3/8%-5/27.

$$BPV \text{ HR} = 0.8072 \times \left( \frac{\$8,558}{\$63.78} \right) = 108.31 \text{ or } 108 \text{ contracts}$$

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Note that the HR = 108 is significantly greater than the 75 contracts suggested by reference to the conversion factor (0.7455) of the 2-3/8%-5/27 security. This is due to the fact that the CTD security carries a relatively short duration of 6.437 years compared to the duration associated with the hedged security of 8.662 years.

It is no coincidence that the ratio of durations is roughly equal to the ratio between the BPV and CF hedge ratios or  $(6.437 \div 8.662) \sim (75 \div 108)$ . I.e., the futures contract is pricing or tracking or correlating most closely with a shorter duration security. Consequently, futures prices will react rather mildly to fluctuating yields. Therefore, one requires more futures to enact an effective hedge.

E.g., what would our hedge ratio be if the CTD security was the on-the-run 2-1/4%-8/27 with a rather longer duration of 8.958 years? This security has a BPV of \$86.99 per \$100,000 face value and a conversion factor for delivery vs. December 2017 Ten-year T-note futures of 0.7314. Our analysis suggests that one might hedge with 73 contracts per \$10 million face value of 2-1/4%-8/27.

$$BPV\ HR = 0.7314 \times \left( \frac{\$8,699}{\$86.99} \right) = 73.14 \text{ or } 73 \text{ contracts}$$

Note that this hedge ratio of 73 contracts is significantly less than the 108 contracts suggested by our analysis above and reasonably similar to the 75 contracts suggested by the CF hedge ratio. This can be explained by the fact that the 2-1/4%-8/27 has pricing characteristics that are quite similar to 2-3/8%-5/27 security which is the subject of the hedge. In particular, the 2-1/4%-8/27 had a duration of 8.958 years which is reasonably close to the 8.662 duration of the 2-3/8%-5/27. Because of the similar risk characteristics of the CTD and hedged security, the CF may do a reasonable job of identifying an appropriate hedge ratio.

## Crossover Risks

This further suggests that, if there is a crossover in the CTD from a short duration security to a longer duration security, the number of futures needed to hedge against the risk of declining prices is decreased. This may be a favorable circumstance for the hedger who is long cash Treasuries and short futures in a ratio prescribed by the BPV technique.

Consider that as prices decline and longer duration securities become CTD, one is essentially over-hedged in a declining market. If on the other hand, prices advance and even shorter duration securities become CTD, the appropriate hedge ratio will tend to increase. Thus, the long hedger becomes under-hedged in a rising market.

Another way of saying this is that there is a certain degree of “convexity” inherent in the relationship that favors the long hedger or long basis trader (long cash and short futures). Conversely, this convexity tends to work to the disadvantage of the short hedger or short basis trader (short cash and long futures).

Once again, we may liken the basis to an option to the extent that option premiums are also affected by convexity. Further, because the long basis trader effectively owns the option, he pays an implicit premium in the difference between prevailing short-term yields and the return on the basis trade as might be simulated in the absence of any CTD crossovers.

The short basis trader is effectively short an option and receives this implicit premium. This implicit premium is reflected in a comparison of the Implied Rate of Return (IRR) relative to prevailing short-term rates.

Note that the BPV of a debt security is dynamic and subject to change given fluctuating yields. As a general rule, BPV declines as a function of maturity; and, as yields increase (decrease), BPVs decline (advance). This implies that the hedge ratio is likewise dynamic. Over a limited period of time, however, HRs may be reasonably stable, provided there is no crossover in the cheapest-to-deliver. As a general rule in practice, it would be commonplace for hedgers to re-valuate and readjust the hedge if rates were to move by perhaps 20-25 basis points.

## Portfolio Hedging

Thus far, our discussion has centered about comparisons between a single security and a Treasury futures contract, a “micro” hedge if you will. But it is far more commonplace for an investor to become concerned about the value of a portfolio of securities rather than focus on a single item within a presumably diversified set of holdings.

How might one address the risks associated with a portfolio of securities, i.e., how to execute a “macro” hedge? The same principles apply whether hedging a single security or a portfolio of securities. Thus, we need to evaluate the risk characteristics of the portfolio in terms of its BPV and duration just as we would examine an individual security. Then we may simply apply the BPV hedge ratio for these purposes.

$$HR = BPV_{portfolio} \div \left( \frac{BPV_{ctd}}{CF_{ctd}} \right)$$

E.g., assume that you held a \$100 million fixed income portfolio with a BPV = \$80,000 and a duration of 8 years. This duration is similar to the duration associated with securities deliverable against the 10-year T-note futures contract, suggesting use of the 10-year as a hedge vehicle. As of October 10, 2017, the CTD security was the 2-3/8%-8/24 with a BPV = \$63.78 per \$100,000 face value unit and a CF = 0.8072. Our analysis suggests that one might sell 1012 futures to hedge the portfolio.

$$HR = \$80,000 \div \left( \frac{\$63.78}{0.8072} \right) = 1012.48 \text{ or } 1012 \text{ contracts}$$

Thus far, our examples illustrated situations where we had effectively hedged individual securities or portfolios in their entirety. In the process, we might effectively push the risk exposure down to near \$0 as measured by BPV or 0 years as measured by duration. But it would actually be uncommon to see an asset manager adjust an actual fixed income risk exposure all the way down to zero.

Asset managers generally measure their performance by reference to a designated “benchmark” or “bogey.” The benchmark is often identified as an index of fixed income securities such as the Barclays U.S. Aggregate Bond Index or some other commonly available measure.

The returns on this benchmark may be identified as the “core” or “beta” returns associated with the portfolio. In addition, the asset manager may exercise some limited degree of latitude in an attempt to outperform the benchmark, or to capture some excess return known as “alpha” in current investment parlance.

Asset managers may be authorized to adjust the duration of the portfolio upwards by a limited amount in anticipation of rate declines and price advances. Or, to adjust duration downwards by a limited amount in anticipation of rate advances and price declines. The following formula provides the appropriate hedge ratio for these operations.

$$HR = \left( \frac{D_{target} - D_{current}}{D_{current}} \right) \times \left[ BPV_{portfolio} \div \left( \frac{BPV_{ctd}}{CF_{ctd}} \right) \right]$$

Where  $D_{target}$  is the target duration;  $D_{current}$  is the current duration.

E.g., let’s return to our example of a \$100 million fixed income portfolio. Assume that the portfolio duration of 8 years was designed to coordinate with the duration of the designated benchmark. Thus, the portfolio manager may be authorized to adjust portfolio duration between 6 and 10 years in pursuit of “alpha.” The asset manager is now concerned about the prospects for rate advances and wishes downwardly to adjust duration from 8 to 6 years. Our analysis suggests that this may be accomplished by selling 253 futures.

$$HR = \left( \frac{6 - 8}{8} \right) \times \left[ \$80,000 \div \left( \frac{\$63.78}{0.8072} \right) \right] \\ = -253.1 \text{ or } \textit{sell 253 contracts}$$

The application of this formula provides asset managers with a great deal of flexibility to adjust the portfolio duration – either upward or downward – to meet the demands of the moment.

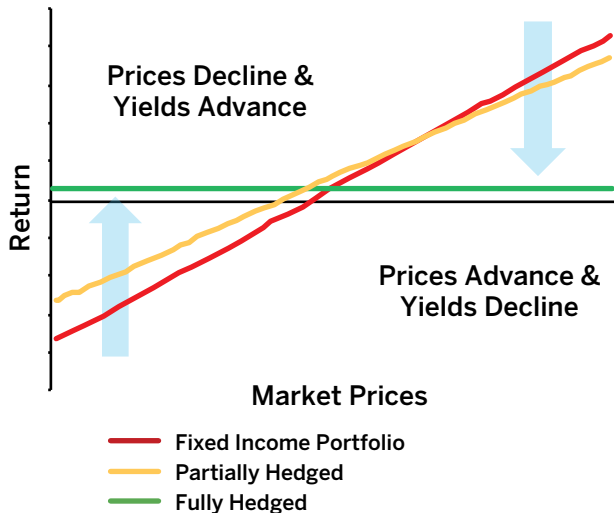
## Bullets and Barbells

Typically, one looks to hedge a Treasury portfolio with the use of Treasury futures which correspond most closely in terms of duration to the average weighted portfolio duration.

E.g., if one held a portfolio with an average weighted duration of 4 years, it would be natural to look to 5-year Treasury note futures as a suitable risk layoff vehicle. If the portfolio had an average weighted duration of 8 years, it would be natural to look to either 10-year Treasury note futures or the recently launched Ultra 10-year Treasury note futures, which tracks one of the three most recently auctioned 10-year notes.

This analysis would tend to work well when the portfolio is constructed predominantly of securities which were close in terms of their durations to the average portfolio duration. Certainly, if the entire portfolio were populated with a variety of recently issued 5-year T-notes, it would behoove the hedger to utilize 5-year Treasury note futures as a hedge, minimizing basis risk and the need for any subsequent hedge management.

### Hedged with Short Futures



A portfolio constructed in such a manner might be labeled a “bullet” portfolio to the extent that it contains reasonably homogeneous securities in terms of maturity and presumably coupon. Under these circumstances, one might simply “stack” the entire hedge in a single Treasury futures contract which most closely conforms to the duration of the portfolio constituents.

Of course, one may attempt to introduce a certain speculative element into the hedge by using longer- or shorter-term futures contracts as the focus of the hedge.

If the yield curve were expected to steepen, a hedge using longer-term futures, e.g., 10- or 30-year Treasury futures rather than 5-year futures, would allow one to capitalize on movement in the curve beyond simply immunizing the portfolio from risk. If the yield curve is expected to flatten or invert, a hedge using shorter-term futures, e.g., 2-year or 3-year Treasury futures rather than 5-year futures, could likewise provide yield enhancement.

But a portfolio need not necessarily be constructed per the “bullet” approach. Consider a portfolio with a duration of 4 years that is constructed using a combination of 2- and 10-year notes and no 5-year notes whatsoever.

A portfolio constructed in such a manner may be labeled a “barbell” portfolio to the extent that it is “weighted” with two extreme duration securities with no intermediate duration securities at all. If one were to simply stack the hedge into 5-year Treasury note futures, the investor becomes exposed to the risk that the shape of the yield curve becomes distorted such that 5-year yields sag below yields in the 2- and 10-year sectors of the curve.

The holder of a barbell portfolio might instead attempt to utilize a combination of various tenured Treasury futures which is weighted with an eye to the proportion of the portfolio devoted to each sector of the yield curve. As such, the hedger may insulate from the risks that the shape of the yield curve will shift.

Thus, an asset manager might categorize his holdings into various sectors of the curve corresponding to available Treasury futures “buckets,” i.e., 2-, 5-, 10- and 30-year securities. Then, the asset manager may calculate the BPV HRs applicable to each of those bucketed portfolios and essentially hedge each element separately.

If, however, the investor wished to introduce a speculative element into the hedge, the use of longer- or shorter-maturity Treasuries driven by an expectation of a steepening or flattening yield curve, respectively, may be in order.

### Comparing Returns of Cash to Futures & Repo

Investors are frequently faced with the choice of allocating funds to Treasury futures or notes. Based upon a similar analysis for an earlier period, we compared returns of investing in CTD Treasury notes to earning income by investing proceeds from the sale of the CTD notes at overnight repo rates and establishing long 10-year Treasury futures (Globex: ZN, Bloomberg: TY) positions.

In our analysis, we assumed that investors either:

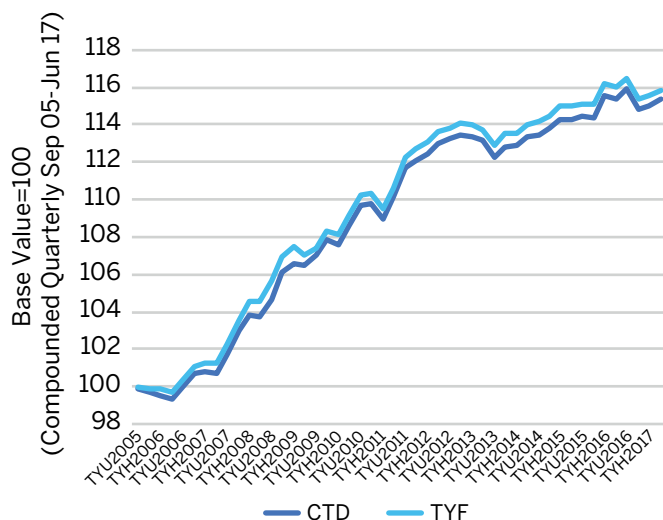
- Maintained long positions in Treasury notes that were CTD for the 10-year Treasury note futures or
- Sold those CTD notes and invested proceeds from the sale at overnight rates, which were compounded for each applicable holding period, and purchased the equivalent long position via Treasury note futures.

We examined quarterly returns of the long CTD Treasury notes position and the long Treasury futures and repo positions from the September 2005 contract month through the June 2017 contract month, 48 quarters. We determined that the combined 10-year Treasury futures and repo positions produced average annual returns of 4.85%, compared to 4.71% for CTD Treasury notes. As a result, the futures and repo positions produced average annual returns 14 basis points higher than the CTD note positions.

The chart below depicts returns for the CTD notes (dark blue line) and futures + repo positions (light blue line) compounded quarterly for the 12-year period.

### Comparing Returns of CTD Treasury Notes (CTD) to 10-Year Futures and Repo (TYF)

#### Compounded Quarterly-September 2005 through June 2017 Contracts



**Table 1: The Delivery Timetable for Treasury Futures\***

(All times refer to Chicago time.)

|                                 | Short Clearing Firm   | CME Clearing  | Long Clearing Firm  |
|---------------------------------|---|---|---|
| <b>First Position Day</b>       |   |   | By 8:00 pm, two business days prior to the first day allowed for deliveries into an expiring futures contract (ie, first day of delivery month), clearing firms report to CME Clearing all open long positions, grouped by account origin (customer or house) and position vintage date.                          |
| <b>Day 1:<br/>Intention Day</b> | By 6:00 pm, the short clearing firm notifies CME Clearing that it intends to make delivery on an expiring contract. Once CME Clearing has matched the short clearing firm to the long clearing firm(s) for delivery, this declaration cannot be reversed.   | At 8:00 pm, CME Clearing matches the delivering short clearing firm to the clearing firm(s) with long positions having the oldest vintage date(s), and then informs the short (long) party that the opposite party will take (make) delivery. | By 8:00 pm, clearing firms report to CME Clearing all open long positions in the expiring futures contract, grouped by account origin (customer or house) and position vintage date.  |
| <b>Day 2:<br/>Notice Day</b>    | By 2:00 pm (3:00 pm on Last Notice Day), using calculations based on the expiring contract's Intention Day settlement price, the short clearing firm must confirm invoice details with CME Clearing.  | At 4:00 pm, CME Clearing runs invoices and provides them to long clearing firm(s) matched to take delivery from the short clearing firm making delivery.  | By 4:00 pm, the long clearing firm assigned to take delivery provides the name and location of its bank to the short clearing firm making delivery.   |
| <b>Day 3:<br/>Delivery Day</b>  | Short and long clearing firms have until 9:30 am to resolve invoice differences. By 10:00 am, the short clearing firm deposits Treasury securities for delivery into its bank account, and it instructs its bank to transfer the securities, via Fed wire, to the long clearing firm's account no later than 1:00 pm. |   | By 7:30 am, the long clearing firm makes funds available, and notifies its bank to remit the funds upon accepting Treasury securities. By 1:00 pm, the long clearing firm's bank has accepted the Treasury securities and has remitted the invoice amount via Fed wire to the short clearing firm's bank account. |

\* Operational details are subject to change, insofar as CME Clearing periodically reviews the physical delivery process and, when necessary, modifies it to enhance its efficiency. For current information, please consult CBOT Rules.

Table 2: Treasury Futures Contracts Summary

|                                      | 2-Year T-Note  | 3-Year T-Note  | 5-Year T-Note   | 10-Year T-Note  | Ultra 10-Year T-Note  | Classic T-Bond  | Ultra T-Bond   |
|--------------------------------------|--|--|---|---|---|---|--|
| <b>Contract Size (\$ Face Value)</b> | 200,000  | 200,000  | 100,000   | 100,000   | 100,000   | 100,000   | 100,000  |
| <b>Deliverable Grade</b>             | Treasury notes. Original term to maturity: not more than 5 years 3 months. Remaining term to maturity: at least 1 year 9 months and not more than 2 years.   | Treasury notes. Original term to maturity: not more than 7 years. Remaining term to maturity: at least 2 years 9 months and not more than 3 years. | Treasury notes. Original term to maturity: not more than 5 years 3 months. Remaining term to maturity: at least 4 years 2 months. | Remaining term to maturity of at least 6 years 6 months, but less than eight years. | Treasury notes. Remaining term to maturity: at least 9 years 5 months and not more than 10 years. | Treasury bonds. Remaining term to maturity: at least 15 years and less than 25 years. | Treasury bonds. Remaining term to maturity: at least 25 years. |
| <b>Delivery Months</b>               | The first three consecutive contracts in the March, June, September, and December quarterly cycle.   |  |   |   |   |   |  |
| <b>Delivery Method</b>               | Physical delivery of contract grade US Treasury securities via the Federal Reserve book-entry wire-transfer system. Delivery invoice price equals the futures contract settlement price, times the size of the futures contract price point, times a conversion factor, plus accrued interest from the delivered security's last coupon payment date to the futures contract delivery date. The conversion factor, computed and published by the Exchange, represents the price of \$1 face value of the delivered security to yield 6 percent per annum as of the first day of the futures contract delivery month. |  |   |   |   |   |  |
| <b>Trading Hours</b>                 | CME Globex: 5:00 pm - 4:00 pm, Chicago time, Sunday – Friday<br>Trading in an expiring contract ceases at 12:01 pm, Chicago time, on the contract's last trading day.  |  |   |   |   |   |  |
| <b>Last Trading Day</b>              | Last business day of the delivery month  |  |   | Seventh business day preceding the last business day of the delivery month          |   |   |  |
| <b>Last Delivery Day</b>             | Third business day following the Last Trading Day  |  |   | Last business day of the delivery month   |   |   |  |
| <b>Price Quote</b>                   | Points (\$2,000) and eighths of 1/32 of a point.   |  | Points (\$1,000) and quarters of 1/32 of a point.   | Points (\$1,000) and halves of 1/32 of a point.                                     |   | Points (\$1,000) and 1/32 of a point.   |  |
| <b>CME Globex Ticker Symbol</b>      | ZT   | Z3N  | ZF  | ZN  | TN  | ZB  | UB   |
| <b>Clearing Code</b>                 | 26   | 3YR  | 25  | 21  | TN  | 17  | UBE  |

**Table 3: December 2017 Ten-Year T-Note Futures Basis (As of October 10, 2017)**

| Securities |            |          |           |            |                | Cheapest to Deliver Analysis |                    |                                  |       |                              |                   |
|------------|------------|----------|-----------|------------|----------------|------------------------------|--------------------|----------------------------------|-------|------------------------------|-------------------|
| Coupon     | Maturity   | Mid      | CUSIP     | Issue      | Delivery Years | Clean Cash Price             | Basis Net of Carry | Treasury Conversion Factor (TCF) | Yield | Futures DV01 (Cash DV01/TCF) | Implied Repo Rate |
| 2.375      | 8/15/2024  | 101.2266 | 912828D56 | 8/15/2014  | 6.67           | 101.2266                     | -0.1185            | 0.8072                           | 2.18% | \$76.51                      | 1.42%             |
| 2.125      | 7/31/2024  | 99.6758  | 912828N9  | 7/31/2017  | 6.58           | 99.6758                      | 0.1633             | 0.7939                           | 2.17% | \$76.56                      | 0.68%             |
| 1.875      | 8/31/2024  | 98.0508  | 912828U3  | 8/31/2017  | 6.67           | 98.0508                      | 0.2718             | 0.7807                           | 2.18% | \$78.02                      | 0.32%             |
| 2          | 6/30/2024  | 98.9336  | 912828X3  | 6/30/2017  | 6.5            | 98.9336                      | 0.358              | 0.7873                           | 2.17% | \$75.87                      | 0.18%             |
| 2.25       | 11/15/2024 | 100.3008 | 912828G38 | 11/17/2014 | 6.92           | 100.3008                     | 1.2816             | 0.7943                           | 2.20% | \$79.37                      | -1.76%            |
| 2.125      | 9/30/2024  | 99.6016  | 912828Y5  | 10/2/2017  | 6.75           | 99.6016                      | 1.6159             | 0.7875                           | 2.19% | \$78.28                      | -2.60%            |
| 2          | 2/15/2025  | 98.4531  | 912828J27 | 2/17/2015  | 7.17           | 98.4531                      | 2.6975             | 0.7741                           | 2.23% | \$82.58                      | -5.15%            |
| 2.125      | 5/15/2025  | 99.1719  | 912828XB1 | 5/15/2015  | 7.42           | 99.1719                      | 3.9209             | 0.7748                           | 2.24% | \$84.79                      | -7.81%            |
| 2          | 8/15/2025  | 98.0938  | 912828K74 | 8/17/2015  | 7.67           | 98.0938                      | 5.2038             | 0.7612                           | 2.26% | \$87.70                      | -10.91%           |
| 2.25       | 11/15/2025 | 99.7813  | 912828M56 | 11/16/2015 | 7.92           | 99.7813                      | 6.2525             | 0.7702                           | 2.28% | \$89.57                      | -12.93%           |
| 1.625      | 2/15/2026  | 94.9063  | 912828P46 | 2/16/2016  | 8.17           | 94.9063                      | 7.9198             | 0.7252                           | 2.30% | \$94.07                      | -17.91%           |
| 1.625      | 5/15/2026  | 94.6641  | 912828R36 | 5/16/2016  | 8.42           | 94.6641                      | 9.0921             | 0.7185                           | 2.31% | \$96.60                      | -20.69%           |
| 1.5        | 8/15/2026  | 93.4414  | 912828A7  | 8/15/2016  | 8.67           | 93.4414                      | 10.3718            | 0.7038                           | 2.32% | \$99.69                      | -24.15%           |
| 2          | 11/15/2026 | 97.2305  | 912828U24 | 11/15/2016 | 8.92           | 97.2305                      | 11.0751            | 0.7307                           | 2.34% | \$100.38                     | -24.51%           |
| 2.25       | 2/15/2027  | 99.1641  | 912828V98 | 2/15/2017  | 9.17           | 99.1641                      | 12.058             | 0.7421                           | 2.35% | \$101.93                     | -26.19%           |
| 2.375      | 5/15/2027  | 100.168  | 912828X88 | 5/15/2017  | 9.42           | 100.168                      | 13.147             | 0.7455                           | 2.35% | \$103.77                     | -28.23%           |
| 2.25       | 8/15/2027  | 99.0391  | 912828R0  | 8/15/2017  | 9.67           | 99.0391                      | 14.4821            | 0.7314                           | 2.36% | \$106.67                     | -31.73%           |

NOTES: December 2017 futures were priced at 125-085 (125-8.5/32nds)

Securities highlighted in red represent least economic-to-deliver; highlighted in green represent most economic-to-deliver.

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Data sources: Bloomberg, CME Group unless otherwise noted.

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