Agenda:

1. Introduction to Barrier Options
2. Why Choose Barrier Options?
3. Types of Barrier Options
4. KIKO Parity

Introduction to Barrier Options

Barrier options are a type of exotic option where the payoff depends not only on the price of the underlying asset at expiration but also on whether the underlying asset's price reaches a specified barrier level during the life of the option. These options are particularly useful in cases where the buyer is only interested in payoffs under certain conditions. This feature makes barrier options more cost-effective than vanilla options, as certain outcomes are excluded.

Why Choose Barrier Options?

Barrier options offer a cost-saving alternative to vanilla options. Since the option may become void or activated when the underlying asset's price crosses a certain barrier, the premium is generally lower compared to a vanilla option with the same strike price and expiration. Investors or firms may choose barrier options to manage costs and focus on particular price ranges, thus enabling more tailored hedging strategies or speculative positions.

Types of Barrier Options

Barrier options can be broadly classified into two categories:

- **Knock-Out Options:** These options become void if the underlying asset crosses a predefined barrier level.
- **Knock-In Options:** These options become active only if the underlying asset crosses a predefined barrier level.

Further, each type of option can be classified based on the position of the barrier:

- **Up-and-Out / Up-and-In:** The barrier level is set above the initial price of the underlying asset.
- **Down-and-Out / Down-and-In:** The barrier level is set below the initial price of the underlying asset.

Example: Vanilla Call Option

Let's start with a simple example of a vanilla call option to illustrate the concept of barrier options.

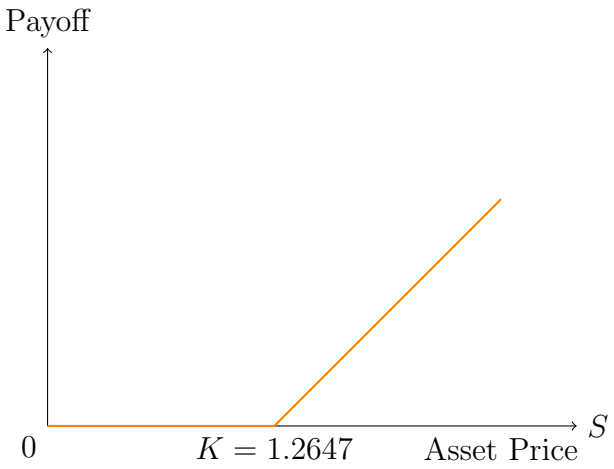
Consider a six-month EUR-USD call option:

$$K = 1.2647 \quad \text{(Strike Price)}$$
$$T = 6 \text{ months}$$

The premium for this call option is USD 2,600. This premium covers the cost of all possible payoffs of the option, even those payoffs that might be unlikely or even impossible due to the range of the underlying asset's price.

Payoff Diagram for Vanilla Call Option

Below is the payoff diagram for a vanilla call option:



The Cost Issue with Vanilla Options

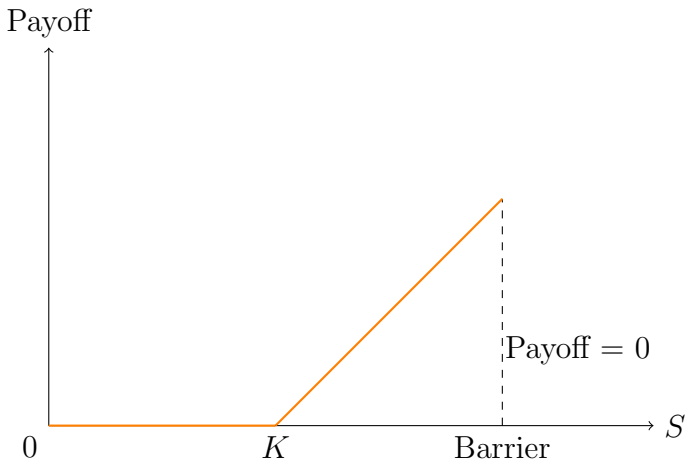
In this vanilla call option, you pay a premium that accounts for all possible payoffs, even if certain high payoffs are unlikely given market conditions or your specific risk appetite.

Barrier Options: Knock-Out Options

Barrier options, such as a **knock-out option**, address this issue by limiting the premium to only certain payoffs, excluding outcomes beyond a chosen price range.

Example of a Knock-Out Barrier Option (Up-and-Out)

Consider an **up-and-out** barrier call option with the following payoff structure:



In this example:

- If the underlying asset price (S) crosses the barrier level during the option's life, the option becomes void, and no payoff is received.
- Since the barrier level is above the strike price, it is called an **up-and-out** barrier option.
- The premium for this option is lower than a vanilla option, as the payoff is only possible up to a specific barrier level.

Down-and-Out Call Option

A **down-and-out call option** is a type of barrier option that becomes void (or "knocked out") if the price of the underlying asset falls below a specified barrier level during the life of the option. This means that the option will not have any payoff at expiration if the underlying asset price breaches the barrier.

Characteristics

- **Barrier Level:** The barrier level is set below the current price of the underlying asset. If the asset price drops below this level at any time before expiration, the option becomes worthless.
- **Premium:** The premium for a down-and-out call option is generally lower than that of a vanilla call option because of the additional condition imposed by the barrier.
- **Payoff at Expiration:** If the barrier is not breached, the payoff at expiration is the same as that of a standard call option, which is defined as:

$$\text{Payoff} = \begin{cases} S_T - K & \text{if } S_T > K \\ 0 & \text{if } S_T \leq K \end{cases}$$

where S_T is the asset price at expiration and K is the strike price.

Example

Consider a down-and-out call option on a stock with the following parameters:

- **Strike Price** $K = 50$
- **Barrier Level** $B = 45$
- **Current Asset Price** $S_0 = 48$
- **Expiration** in 6 months

If the stock price remains above 45 at all times until expiration, the option can still be exercised if $S_T > K$.

Payoff Scenarios

- **Scenario 1:** If $S_T = 55$:

  $$\text{Payoff} = 55 - 50 = 5$$

  The option is exercised.

- **Scenario 2:** If $S_T = 40$ (the barrier is breached):

  $$\text{Payoff} = 0$$

  The option is knocked out and has no value.

- **Scenario 3:** If $S_T = 48$:

  $$\text{Payoff} = 0$$

  The option expires worthless because the asset price is below the strike price.

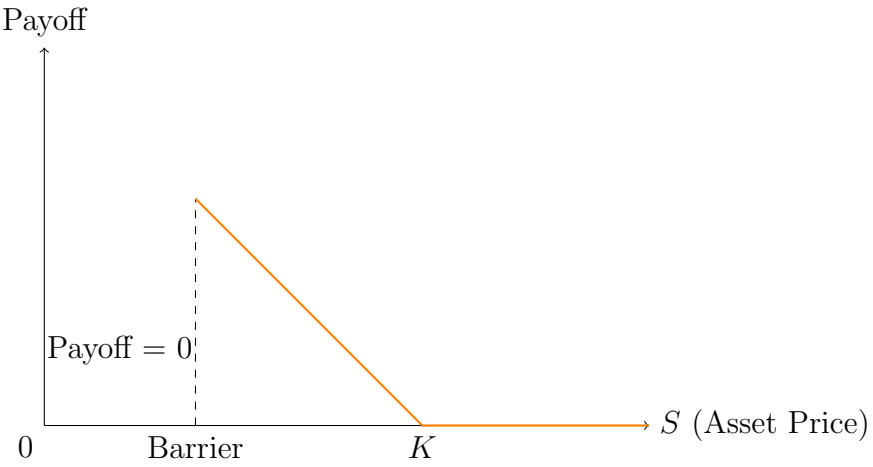
Double Knock-Out Call Option

A **double knock-out call option** has two barriers: one above and one below the current asset price. The payoff is received only if the asset price stays between the two barriers. If the asset price crosses either barrier, the option becomes void.

- This type of option offers even lower premiums since it restricts the range of valid payoffs further.
- Typically, the strike price is set between the two barriers.

Barrier Options on Put Options

For put options, barrier options work similarly but with adjustments for their payoff structure. Below is an example for a down-and-out put option.



Knock-In Options

In **knock-in options**, the option becomes active only if a specific barrier level is reached.

- For instance, in an **up-and-in call option**, the option activates if the asset price rises above a predefined barrier level.

Double Knock-In Barrier Options

In a **double knock-in barrier option**, there are two barriers. The option is activated if the asset price crosses either barrier, providing added flexibility for scenarios where price fluctuations are expected in both directions.

Barrier Options and Barrier Types

Additionally, we can classify them further based on the time intervals in which barriers are considered, specifically into continuous, discrete, and window barriers.

1. Continuous Barrier Options

Continuous barrier options allow the barrier level to be monitored at all times throughout the option's life. This means that any breach of the barrier, regardless of when it occurs, will affect the option's validity.

- **Example:** A continuous down-and-out call option becomes worthless if the price of the underlying asset falls below a specified barrier level at any moment during its life.

- **Payoff Structure:** The payoff is defined as follows:

$$\text{Payoff} = \begin{cases} S_T - K & \text{if } S_T > K \text{ and } S_t > B \text{ for all } t \\ 0 & \text{if } S_t \leq B \text{ at any time } t \end{cases}$$

2. Discrete Barrier Options

Discrete barrier options monitor the barrier level only at specific times, typically at expiration or a set of predetermined points. Breaches that occur outside these designated times do not affect the validity of the option.

- **Example:** A discrete up-and-out call option that is valid as long as the underlying asset's price does not exceed the barrier level at the expiration date.
- **Payoff Structure:** The payoff is given by:

$$\text{Payoff} = \begin{cases} S_T - K & \text{if } S_T > K \text{ and } S_T \geq B \text{ at expiration} \\ 0 & \text{if } S_T < B \end{cases}$$

3. Window Barrier Options

Window barrier options are a hybrid type that incorporates features of both continuous and discrete options. They only consider the barrier during a specific time window within the life of the option. Outside of this window, the barrier is not monitored.

- **Example:** A window down-and-out call option might be valid only during the last month of its life. If the underlying asset's price breaches the barrier during this window, the option becomes void.
- **Payoff Structure:** The payoff can be described as:

$$\text{Payoff} = \begin{cases} S_T - K & \text{if } S_T > K \text{ and } S_t > B \text{ during the window} \\ 0 & \text{if } S_t \leq B \text{ during the window} \end{cases}$$

Conclusion

The distinction between continuous and discrete barrier options, as well as the introduction of window barriers, highlights the flexibility of barrier options in financial markets. Each type serves different investment strategies and risk management needs, allowing traders to tailor their approach based on market conditions and specific goals.

Knock-In and Knock-Out Parity

The concept of parity between knock-in and knock-out options arises from their inherent characteristics and pricing relationships. This parity exists due to the dependence of these options on the same underlying asset and their respective barrier levels.

Parity Definition

Knock-in and knock-out options exhibit a parity relationship expressed as:

$$C_{\text{up-and-in}} = C_{\text{vanilla}} - C_{\text{down-and-out}}$$

where:

- $C_{\text{up-and-in}}$ is the price of the up-and-in call option, which becomes valid only when the asset price exceeds a specified barrier.
- C_{vanilla} is the price of a standard call option, which has no barriers.
- $C_{\text{down-and-out}}$ is the price of the down-and-out call option, which becomes void if the asset price falls below a specified barrier.

The justification is simple:

- Assume you hold both a KI (Knock-In) and KO (Knock-Out) option.
- If the barrier is untouched, the KO option pays a vanilla payoff at expiry.
- If the barrier is touched, the KI option pays a vanilla payoff at expiry.
- Since the payoff is identical to a vanilla option, its price must also be equal due to no-arbitrage.